MARK BOX		
Problem	Points	
1	15	
2	16	
3	16	
4	16	
5	16	
6	16	
*	5	
Total	100	

MATH 550
SPRING 1994
FINAL EXAM

Prof. Girardi

NAME: ______________________________

SSN: ______________________________

Instructions:

(1) To receive credit you must work in a logical fashion, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the line (or in the box) provided.
(2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems. Problem * is your paragraph on Calculus at the Workplace.
(3) Allowed are a calculator and the following class handouts: table of integrals, calculus formula sheet, coordinate systems, and informal summary 1 & 2 (with whatever personal notes you choose to write on them). Not allowed are other notes and books.
(4) This exam covers (from *Intro. to Vector Analysis* by Davis & Snider) sections: 1.1 – 1.12, 1.14, 2.1, 2.2 3.1, 3.3–3.7, 4.1–4.4, 4.8–4.12, 4.15, 4.16.

1. Consider the scalar field $f(x,y,z) = 6xz - xy^2 - z^3$ along with the point $P = (2,0,1)$. Find:

a) $\vec{\nabla} f(x,y,z) = \langle$ ____________ , ____________ , ____________ $\rangle$

b) $\vec{\nabla} f(P) = \langle$ ____________ , ____________ , ____________ $\rangle$

c) the equation of the tangent plane to the surface $f(x,y,z) = 11$ at the point P is

____

d) The directional derivative of f at the point P in the direction of $\vec{v} = \langle 1,0,1 \rangle$ is

____

e) At P, the function f increases most rapidly in the direction of $\langle ......, ..., ... \rangle$.

2. Consider the two parallel planes $\mathcal{P}_1$ and $\mathcal{P}_2$ along with the point $P_0 = (2, 0, 0)$ in the plane $\mathcal{P}_1$ where

(1) $\mathcal{P}_1$ has the equation $2x - y + 2z = 4$

(2) $\mathcal{P}_2$ has the equation $2x - y + 2z = 22$.

a) A parameterization of the line through P_0 that is perpendicular to the plane $\mathcal{P}_1$ is, for $-\infty < t < \infty$:

$\vec{R}(t)$ = ______________________________

b) Find the perpendicular (i.e. shortest) distance between $\mathcal{P}_1$ and $\mathcal{P}_2$.
Hint: use part (a). ANSWER: ____________________

3. Evaluate $\int_{(0,0)}^{(2,1)} \left(15x^4 - 3x^2y^2\right)\, dx - 2x^3y\, dy$ along the path $2x^4 - 6xy^2 - 20y = 0$.

ANSWER: ____________________________

4. Consider the vector field

$$\vec{F}(x, y, z) = \left\langle \, x + \tan\left(e^{y}\right), \; \sqrt{x^{3} + x^{1} + 2}\,, \; \sin\left[(y)(z)(z-4)\right] \, \right\rangle \; .$$

Let S be the complete surface of the region bounded by the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 4$ that consists of:

(1) the top S_1 circle in the $z = 4$ plane
(2) the bottom S_2 circle in the $z = 0$ plane
(3) the curved side S_3 of the cylinder.

Using the Divergence Theorem and a clever method that we saw in class of considering a vector field $\vec{G}$ that is similar to but "nicer" than $\vec{F}$, compute the below integral.

Answer: $\iint_S \vec{F} \cdot \vec{n}\, dS =$ ______________________________

5. Fix two positive numbers a and b. Consider the ellipse , that is the intersection of $x^2 + y^2 = a^2$ and $z = by$. Let $\mathcal{S}$ be the (flat) surface that sits in the $z = by$ plane and that is bounded by , .

a) A parametrization of $\mathcal{S}$ is: $\vec{R}(x, y) = \langle\ x\ ,\ y\ ,$ ________________ $\rangle$ with the following restrictions on x and y:

____ .

b) The surface area of $\mathcal{S}$ is: ______________________________.

c) By means of Stokes' Theorem, find $\int_\Gamma \vec{F} \bullet d\vec{R}$ for the above , and $\vec{F}(x, y, z) = \langle\ x\ ,\ x + y\ ,\ x + y + z\ \rangle$.

Answer: $\int_\Gamma \vec{F} \bullet d\vec{R} =$ ______________________________

6. A puffo is running along a path C given by, for $0 \leqslant t \leqslant 2$,

$$\vec{,}(t) = \left\langle \ (2-t)e^t \ , \ t \ , \ 2t \ \right\rangle \ .$$

Compute $\int_C \vec{G} \bullet d\vec{,}$ where

$$\vec{G}(x,y,z) = \left\langle \ (1+x)e^{x+y} \ , \ xe^{x+y} + 5z \ , \ 7y \ \right\rangle \ .$$

Hint, some cleverness is needed. Is $\vec{G}$ conservative ? If not, can you find a conservative $\vec{F}$ that is similar to $\vec{G}$ and then take advantage to the similarity?

Answer: $\int_C \vec{G} \bullet d\vec{,} =$ ______________________________