MATH 550 EXAM 2 SPRING 1994

MARK BOX		
Problem	Points	
1	30	
2	35	
3	35	
Total	100	

26 March 1994

NAME: ______________________________

SSN: ______________________________

Instructions:

(1) To receive credit you must work in a logical fashion, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the line (or in the box) provided.

(2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) Allowed are a calculator and the class handouts, as indicated on the syllabus. Not allowed are other notes and books.

(4) This exam covers (from *Intro. to Vector Analysis* by Davis & Snider) sections: 2.3, 3.1, 3.3 – 3.7, 4.1.

1. Consider the point $P = (1, 2, 0)$ along with the scalar field

$$f(x, y, z) = 6xz \; - \; ye^z$$

and the vector field

$$\vec{F}(x, y, z) = \langle \; \tan x \; , \; x^2 + 4y + 7z \; , \; 2x + z^3 \; \rangle \; .$$

Find:

a) $\overrightarrow{\nabla} f(x, y, z) = \langle$ ______________ , ______________ , ______________ $\rangle$

b) $\overrightarrow{\nabla} f(1, 2, 0) = \langle$ ______________ , ______________ , ______________ $\rangle$

c) the equation of the tangent plane to the surface $f(x, y, z) = -2$ at the point P is:

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d) The directional derivative of f at the point P in the direction of $\vec{v} = \langle \; 1, 0, 1 \; \rangle$ is:

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e) At P, the function f increases most rapidly in the direction of $\langle$ ......, ..., ... $\rangle$.

f) div $\vec{F} =$ ____

g) $\overrightarrow{\text{curl}}\, \vec{F} =$ ____

2. Consider the vector field

$$\vec{F}(x,y,z) \;=\; \left\langle \frac{y}{x^2+y^2},\; \frac{-x}{x^2+y^2},\; 0 \right\rangle$$

and the curve $\mathcal{C}$ parameterized by, for $-1 \leqslant t \leqslant 1$,

$$\vec{R}(t) \;=\; \left\langle\, t,\; \sqrt{1-t^2},\; 0 \,\right\rangle \;,$$

a) Using **this** parameterization, express the line integral of $\vec{F}$ along $\mathcal{C}$ as an integral with respect to t. The only variable in your integrand should be t.

$$\int_{\mathcal{C}} \vec{F} \bullet d\vec{R} \;=\; \int \underline{\hspace{20em}}\; dt\;.$$

b) Find a re-parameterization $\vec{S}(t)$ of $\vec{R}(t)$ that involves $\cos t$ and $\sin t$.

$$\vec{S}(t) \;=\; \underline{\langle \hspace{14em} \rangle} \qquad \underline{\hspace{2em} \leqslant t \leqslant \hspace{2em}}\;.$$

c) Using the fact that the line integral is independent of the parameterization of $\mathcal{C}$ and using the re-parameterization $\vec{S}$ of $\mathcal{C}$, express the line integral of $\vec{F}$ along $\mathcal{C}$ as integral with respect to t. Again, the only variable in your integrand should be t but this time your integrand should look different from the integrand in part (a).

$$\int_{\mathcal{C}} \vec{F} \bullet d\vec{R} \;=\; \int \underline{\hspace{20em}}\; dt\;.$$

d) Now find the line integral of $\vec{F}$ along $\mathcal{C}$ (i.e. evaluate the above integral of your choice.)

$$\int_{\mathcal{C}} \vec{F} \bullet d\vec{R} \;=\; \underline{\hspace{12em} dt}\;.$$

3. A rectangular sheet-metal tank is open at the top and is filled with 4 cubic meters of water. See the below diagram. Using the method of Lagrange Multipliers, find the dimensions of the tank so that the metal surface in contact with the water is minimal.

Answer: $\underline{x = \qquad\qquad meters}$ $\underline{y = \qquad\qquad meters}$ $\underline{z = \qquad\qquad meters}$