MARK BOX		
Problem	Points	
1	10	
2	10	
3	10	
4	10	
5	15	
6	15	
7	15	
8	15	
Total	100	

MATH 550
SPRING 1993
FINAL EXAM

Prof. Girardi

NAME: _____

SSN: _____

Instructions:

(1) To receive credit, you must work in a logical fashion, show all your work, and either box your answer or (when applicable) put your answer on the line or in the box provided.
(2) Calculators & a formula sheet allowed. Open books & open notes not allowed.
(3) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(4) Write your name on each page.

1. Find the distance between the point $(1, 2, 3)$ and the plane $2x - 2y + z = 4$.

Answer: _____

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2. Determine the arc length L of the curve

$$x = e^t \cos t \qquad y = e^t \sin t \qquad z = 0$$

between $t = 0$ and $t = 1$.

Arc length as an integral: $L =$ _____

Arc length as a number: L = _____

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3. Consider the scalar field $f(x,y,z) = 6x - y^2 - xe^z$ along with the point $P = (1,2,0)$. Find:

a) $\nabla f(x,y,z) = \langle$ _____ , _____ , _____ $\rangle$

b) $\nabla f(1,2,0) = \langle$ _____ , _____ , _____ $\rangle$

c) an equation of the tangent plane to the surface $f(x,y,z) = 1$ at the point P is

_____

d) The directional derivative of f at the point P in the direction of $\mathbf{v} = \langle 1,0,1 \rangle$ is

_____

e) At P, the function f increases most rapidly in the direction of $\langle ....., ..., ... \rangle$.

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4. Fill in the blanks.
Let $\mathbf{F}$ be a vector field that is defined and continuous throughout a domain D.

a) $\mathbf{F}$ is called *conservative* in D if there is a scalar field ϕ defined in D such that

_____ .

In this case, ϕ is called a _____ of $\mathbf{F}$.

b) A continuously differentiable vector field $\mathbf{F}$ in a domain D is conservative if and only if the line integral of $\mathbf{F}$ along every regular curve C in D

_____ .

In this case, if ϕ is a potential of F in D and C goes from points P to Q, then $\int_C \mathbf{F} \cdot d\mathbf{R} =$ _____ .

c) A continuously differentiable vector field $\mathbf{F}$ in a *simply connected* domain D is conservative if and only curl $\mathbf{F} =$ _____ throughout D .
Next, consider the vector field

$$\mathbf{F} = \langle \sin x \ , \ y^2 \ , \ e^z \ \rangle \ .$$

d) The domain D of definition of $\mathbf{F}$ is _____ .

e) Is $\mathbf{F}$ conservative in D? _____ Why or why not?

f) Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$ where C is the helix from $(1, 0, 0)$ to $(1, 0, 4)$ given by $\mathbf{R}(t) = \langle \cos(2\pi\, t), \sin(2\pi\, t), 4t \rangle$. Hint: there is an easy way.....

answer: $\int_C \mathbf{F} \cdot d\mathbf{R} =$ _____ .

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5. Compute the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS$ where $\mathbf{F} = \langle x, 0, 0 \rangle$ over the triangle with vertices $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 3)$, taking the normal pointing away from the origin.

   Answer: $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS =$ _____

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6. Find the surface area S of the surface given parametrically by

$$x = s^2 \qquad y = -\sqrt{2}\,st \qquad z = t^2$$

where $0 \le s \le 1$ and $0 \le t \le 2$.

Surface area as an integral: $S =$ _____

Surface area as a number: S = _____

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7. Consider the two vector fields

$$\mathbf{F} = \left\langle\, x + \tan\left(e^{y}\right),\ \sqrt{x^3 + x^1 + 2}\,,\ \sin\left[(y)(z)(z-2)\right] \right\rangle$$

$$\mathbf{G} = \left\langle\, x + \tan\left(e^{y}\right),\ \sqrt{x^3 + x^1 + 2}\,,\ 0 \right\rangle\ .$$

Let S be the complete surface of the region bounded by the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 2$ which consists of:

(1) the top S_1 circle in the $z = 2$ plane
(2) the bottom S_2 circle in the $z = 0$ plane
(3) the curved side S_3 of the cylinder.

a) Without doing computations, explain why $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$ and $\iint_S \mathbf{G}\cdot\mathbf{n}\,dS$ are equal.

b) Use part (a) and the Divergence theorem to compute the common value of the above two integrals.

Answer: $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS =$ _____

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8. A Volvo is running along the branch of the curve $xy = 1$ that lies in the first quadrant in such a way that its abscissa (ie. the x coordinate) is increasing in time. The Volvo's speed (i.e. *scalar* velocity) equals 55 miles per hour. Express the Volvo's velocity vector as a function of the abscissa x .

   ANSWER: $\mathbf{v}(x) = <$ _____ , _____ $>$

   Hint: The velocity vector is a scalar multiple of the unit tangent to the curve. So begin by finding the unit tangent to the curve.