MARK BOX		
Problem	Points	You
1	20	
2	20	
3	30	
4	30	
Total	100	

MATH 550 SPRING 1993 EXAM 2

NAME: ______________________________

SSN: ______________________________

Instructions:

(1) To receive credit, you must work in a logical fashion, show all your work, and either box your answer or (when applicable) put your answer on the line or in the box provided.
(2) Calculators allowed. "Formula sheets", open books, open notes are not allowed.
(3) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.

1. Fill in the blank ...
Let $\mathbf{F}$ be a vector field that is defined and continuous throughout a domain D.

a) $\mathbf{F}$ is called *conservative* in D if there is a scalar field ϕ defined in D such that

____ .

In this case, ϕ is called a ______________________ of $\mathbf{F}$.

b) A continuously differentiable vector field $\mathbf{F}$ in a domain D is conservative if and only if the line integral of $\mathbf{F}$ along every regular curve C in D

____ .

In this case, if ϕ is a potential of F in D and C goes from points P to Q, then $\int_C \mathbf{F} \cdot d\mathbf{R} =$ ______________________ .

c) A continuously differentiable vector field $\mathbf{F}$ in a *simply connected* domain D is conservative if and only curl $\mathbf{F} =$ ____________ throughout D .

2. Consider the vector field

$$\mathbf{F} = \left\langle \frac{2x}{x^2+y^2} \, , \, \frac{2y}{x^2+y^2} \, , \, 2z \right\rangle .$$

a) The domain D of definition of $\mathbf{F}$ is

____ .

b) Can you use the "curl test" (i.e. part 1c from this test) to determine whether $\mathbf{F}$ is conservative in D? ______________ Why or why not?

c) Find a potential ϕ of $\mathbf{F}$. ANSWER: $\phi =$ ______________________________ .

3. Consider the vector field

$$\mathbf{F} = \langle\ \sin x\ ,\ y^2\ ,\ e^z\ \rangle\ .$$

a) The domain D of definition of $\mathbf{F}$ is ____________________________ .

b) Is $\mathbf{F}$ conservative in D ? _______________ Why or why not?

c) Find the line integral $\int_C \mathbf{F} \cdot d\mathbf{R}$ where C is the helix from $(1,0,0)$ to $(1,0,4)$ given by $\mathbf{R}(t) = \langle \cos(2\pi\, t), \sin(2\pi\, t), 4t \rangle$. Hint: there is an easy way.....

answer: $\int_C \mathbf{F} \cdot d\mathbf{R} =$ ____ .

4. Consider the paraboloid of revolution $z = x^2 + y^2$ for $0 \leq z \leq 4$.

a) A parametric representation of this paraboloid is

$$R(\theta, t) = \langle \, \underline{\hspace{4cm}} \, , \, \underline{\hspace{4cm}} \, , \, \underline{\hspace{4cm}} \, \rangle \; .$$

b) Express the surface area SA of this paraboloid as a double integral. Indicate the limits of integration but do NOT integrate.

$$SA = \int \int \qquad\qquad\qquad\qquad\qquad\qquad d\theta \, dt \; .$$