

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
TOTAL	60	
%	100	

NAME: \_\_\_\_\_

SSN: \_\_\_\_\_

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**INSTRUCTIONS:**

- To receive credit you must:
    - work in a logical fashion, show all your work, indicate your reasoning
    - when applicable put your answer on/in the line/box provided
    - if no such line/box is provided, then box your answer
    - if you use your calculator on a particular problem, then indicate so.
  - The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
  - As indicated on the syllabus:
    - allowed is a calculator (but not a computer)
    - allowed are the class handouts: table of integrals, calculus formula sheet, and Spring 2000 informal summary (along with your personal scribbles on them)
    - not allowed are books and other notes.
  - During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
  - This exam covers (from *Vector Calculus* by Marsden&Tromba, 4<sup>th</sup> ed.): Ch. 1, Ch. 2, Section 3.1, Chs. 4 – 7 .
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**1a.** Figure 4.4.9 from the text, which is shown on the first page of this exam, shows some flow lines and moving regions for a fluid moving in the plane field velocity field  $\vec{V}$ .

Where is  $\text{div } \vec{V} > 0$ ? \_\_\_\_\_ .

Where is  $\text{div } \vec{V} < 0$ ? \_\_\_\_\_ .

Intuitively explain your answers in (a few) complete sentences.

**1b.** Figure 4.4.7 from the text, which is shown on the first page of this exam, shows the movement of a small rigid paddle wheel that is placed in moving fluid. The fluid has velocity field  $\vec{V}$ .

What can you say about the  $\text{curl } \vec{V}$ ? \_\_\_\_\_ .

Intuitively explain your answer in (a few) complete sentences.

**2.** Let  $\mathbf{P}$  be the parallelogram with vertices:

$$p_1 = (1, 2) \quad , \quad p_2 = (-1, 5) \quad , \quad p_3 = (2, 6) \quad , \quad p_4 = (4, 3) .$$

Let  $\mathbf{S}$  be the unit square, this it is a parallelogram with vertices:

$$s_1 = (0, 0) \quad , \quad s_2 = (1, 0) \quad , \quad s_3 = (1, 1) \quad , \quad s_4 = (0, 1) .$$

**2a.** A one-to-one and onto linear transformation  $T: \mathbf{S} \rightarrow \mathbf{P}$  that satisfies

$T(s_i) = p_i$  for each  $i = 1, 2, 3, 4$  is:

$$T(u, v) = \langle \underline{\hspace{4cm}} \quad , \quad \underline{\hspace{4cm}} \rangle .$$

**2b.** The Jacobian Matrix  $J_T$  of T is  $\underline{\hspace{4cm}}$  .

**2c.** The determinate of  $J_T$  is  $\underline{\hspace{4cm}}$  .

**2d.** Using only the answer from 2c and the fact that the area of  $\mathbf{S}$  is 1, one of the key ideas from Chapter 6 tells us that: (hint: IS 6.4.2 ★)

the area of  $\mathbf{P}$  is:  $\underline{\hspace{4cm}}$  .

**3.** Let  $a, b > 0$  and

$$\mathbf{D} = \left\{ (x, y) : \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \leq 1 \right\}$$
$$\mathbf{D}^* = \{ (u, v) : u^2 + v^2 \leq 1 \} .$$

So  $\mathbf{D}$  is an ellipse (inside and boundary included) and  $\mathbf{D}^*$  is the unit circle (inside and boundary included). Let  $\mathbf{A}$  be the area of  $\mathbf{D}$ .

**3a.** A 1-to-1 transformation  $T$  so that  $T(\mathbf{D}^*) = \mathbf{D}$  is:

$$T(u, v) = \langle \text{_____} , \text{_____} \rangle .$$

**3b.** Expressed as a double integral over  $\mathbf{D}^*$ ,

$$\mathbf{A} = \text{_____} .$$

**3c.** By integrating the integral in **3b** (and perhaps using the fact that the area of  $\mathbf{D}^*$  is  $\pi$ ), we can compute that  $\mathbf{A} = \text{_____}$ .

4. Let  $\mathcal{V}$  be the solid in the first octant bounded by the plane

$$x + 2y + z = 6 .$$

Make a (rough) sketch of  $\mathcal{V}$ . The volume  $\mathbf{V}$  of  $\mathcal{V}$  can be expressed as the following triple integrals:

$$\mathbf{V} = \int \int \int dz dx dy$$

$$\mathbf{V} = \int \int \int dx dz dy .$$

You do NOT need to actually perform the integration.

5. The temperature inside the capsule bounded by

$$z = 9 - x^2 - y^2 \quad \text{and} \quad z = 3x^2 + 3y^2 - 16$$

varies from point to point as

$$T(x, y, z) = z(x^2 + y^2) .$$

The average temperature of the capsule is (exactly, not approx.): \_\_\_\_\_ .

Hint: do a change of variables to cylindrical coordinates.

More space for this problem on the next page ...

**More space for the last problem:**