Prof. Girardi	Math 550	25 March	Spring 2000	Exam 2

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
TOTAL	50	
%	100	

NAME: ____________________

SSN: ______________________

INSTRUCTIONS:

1. To receive credit you must:
   (a) work in a logical fashion, show all your work, indicate your reasoning
   (b) when applicable put your answer on/in the line/box provided
   (c) if no such line/box is provided, then box your answer
   (d) if you use your calculator on a particular problem, then indicate so.
2. The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
3. As indicated on the syballus:
   (a) allowed is a calculator (but not a computer)
   (b) allowed are the class handouts: table of integrals, calculus formula sheet, and Spring 2000 informal summary (along with your personal scribbles on them)
   (c) not allowed are books and other notes.
4. During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
5. This exam covers (from *Vector Calculus* by Marsden&Tromba, 4^{th} ed.): § 4.3, 4.4 and Chs. 5, 6 .

1a. Figure 4.4.9 from the text, which is shown on the first page of this exam, shows some flow lines and moving regions for a fluid moving in the plane field velocity field $\vec{\mathbf{V}}$.

Where is div $\vec{\mathbf{V}} > 0$? ______________________________ .

Where is div $\vec{\mathbf{V}} < 0$? ______________________________ .

Intuitively explain your answers in (a few) complete sentences.

1b. Figure 4.4.7 from the text, which is shown on the first page of this exam, shows the movement of a small rigid paddle wheel that is placed in moving fluid. The fluid has velocity field $\vec{\mathbf{V}}$.

What can you say about the $\overrightarrow{\text{curl}}\ \vec{\mathbf{V}}$? ______________________ .

Intuitively explain your answer in (a few) complete sentences.

2. Let **P** be the parallelogram with vertices:

$$p_1 = (1,2) \quad , \quad p_2 = (-1,5) \quad , \quad p_3 = (2,6) \quad , \quad p_4 = (4,3) \ .$$

Let **S** be the unit square, this it is a parallelogram with vertices:

$$s_1 = (0,0) \quad , \quad s_2 = (1,0) \quad , \quad s_3 = (1,1) \quad , \quad s_4 = (0,1) \ .$$

2a. A one-to-one and onto linear transformation $T\colon \mathbf{S} \to \mathbf{P}$ that satisfies $T(s_i) = p_i$ for each $i = 1,2,3,4$ is:

$T(u,v) \ = \ \langle$ ________________________ , ________________________ $\rangle$.

2b. The Jacobian Matrix J_T of T is _____________________________ .

2c. The determinate of J_T is _______________ .

2d. Using only the answer from 2c and the fact that the area of **S** is 1, one of the key ideas from Chapter 6 tells us that: (hint: IS 6.4.2 ★)

the area of **P** is: ____ .

3. Let $a, b > 0$ and

$$\mathbf{D} = \left\{ (x,y) \colon \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 \leq 1 \right\}$$
$$\mathbf{D}^* = \left\{ (u,v) \colon u^2 + v^2 \leq 1 \right\} .$$

So $\mathbf{D}$ is an ellipse (inside and boundary included) and $\mathbf{D}^*$ is the unit circle (inside and boundary included). Let $\mathbf{A}$ be the area of $\mathbf{D}$.

3a. A 1-to-1 transformation T so that $T(\mathbf{D}^*) = \mathbf{D}$ is:

$T(u,v) = \langle$ ____________________ , ____________________ $\rangle$.

3b. Expressed as a double integral over $\mathbf{D}^*$,

$\mathbf{A} =$ ____ .

3c. By integrating the integral in **3b** (and perhaps using the fact that the area of $\mathbf{D}^*$ is π), we can compute that $\mathbf{A} =$ ________________.

4. Let $\mathcal{V}$ be the solid in the first octant bounded by the plane

$$x + 2y + z = 6 \ .$$

Make a (rough) sketch of $\mathcal{V}$. The volume $\mathbf{V}$ of $\mathcal{V}$ can be expressed as the following triple integrals:

$$\mathbf{V} \ = \int \qquad\qquad \int \qquad\qquad \int \qquad\qquad dz\,dx\,dy$$

$$\mathbf{V} \ = \int \qquad\qquad \int \qquad\qquad \int \qquad\qquad dx\,dz\,dy \ .$$

You do NOT need to actually perform the integration.

5. The temperature inside the capsule bounded by

$$z = 9 - x^2 - y^2 \qquad \text{and} \qquad z = 3x^2 + 3y^2 - 16$$

varies from point to point as

$$T(x, y, z) = z(x^2 + y^2)\ .$$

The average temperature of the capsule is (exactly, not approx.): __________ .
Hint: do a change of variables to cylindrical coordinates.

More space for this problem on the next page ...

More space for the last problem: