A Transition to Advanced Mathematics by Smith, et. Al (7th edition)

Definitions:

- **Indirect proofs:** the techniques that are based on tautologies that replace the statement to be proved by an equivalent statement of statements.
- **Example of a contrapositive proof**
  - (P ==> Q) <==> (~Q => ~P)
    - Since both of those are equivalent statements, we first give a proof of ~Q => ~P and then conclude by replacing that with P => Q.
      - PROOF OF CONTRAPOSITION OF P => Q
        Assume ~Q
        Therefore, ~P
        Thus, ~Q => ~P
        Therefore, P => Q
- **Proof by Contradiction**
  - A proof that if a statement cannon be false, then it must be true.
    - PROOF BY CONTRADICTION
      Suppose ~P.
      Therefore, Q
      Therefore, ~Q
      Hence, Q ∧ ~Q a contradiction
      Thus, P.

Proofs by contradiction can be applied to any proposition P, whereas direct proofs and proofs by contraposition can be used only for conditional sentences.

- **Parity: the attribute of being either odd or even**
- **Consistent axiom systems: statements that neither the negation nor the statement can be proved.**
- **Undecidable:** truth is independent of the truth of the axioms.

- EXAMPLES (in book) p .40

  -Let m be an integer. Prove that if m^2 is even, then m is even.

  { P="m^2 is even" Q= "m is even"}

  Proof: suppose that the integer m is not even. (~Q) then m is odd so m=2k+1 for some integer k. then:

  $$m^2 = (2k+1)^2 = 4k^2 +4k +1 = 2(2k^2$$

Since m^2 is twice an integer, plus 1, m^2 is odd. Therefore, m^2 is not even.

Thus, if m is not even, then m^2 is not even. By contraposition, if m^2 is even, then m is even.