# PRINCIPLE OF MATHEMATICAL INDUCTION
# (PMI)

---

Let $\mathbb{N} = \{1, 2, 3, \dots\}$ be the natural numbers.
Let $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ be the integers.
Let $P(n)$ be a statement (that is either true or false) about $n$.
Sometimes we denote $P(n)$ by $P_n$.

---

## § 2.4: PMI (basic form)

If

BASE STEP: $P(1)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{N}$: $[\ P(n) \text{ is true }] \implies [\ P(n+1) \text{ is true }]$

then $P(n)$ is true for each $n \in \mathbb{N}$.

## § 2.4: PMI (generalized form) (also known as: doesn't matter where you start form)

Fix $n_0 \in \mathbb{Z}$.
If

BASE STEP: $P(n_0)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{Z}$ with $n \geq n_0$: $[\ P(n) \text{ is true }] \implies [\ P(n+1) \text{ is true }]$

then $P(n)$ is true for each $n \in \mathbb{Z}$ such that $n \geq n_0$.

## § 2.5: PMI (strong form) (our book calls this PCI = Principle of Complete Induction)

Fix $n_0 \in \mathbb{Z}$.
If

BASE STEP: $P(n_0)$ is true

INDUCTIVE STEP: for each $n \in \mathbb{Z}$ with $n \geq n_0$:
$[\ P(j) \text{ is true for } j = n_0, 1 + n_0, \dots, n\ ] \implies [\ P(n+1) \text{ is true }]$

then $P(n)$ is true for each $n \in \mathbb{Z}$ such that $n \geq n_0$.