Division Algorithm

Division Algorithm for $\mathbb{N}$

Remark. Think about taking an $n \in N$ and dividing it by $d \in \mathbb{N}$. What happens?

Let's look at an example: take $n = 11$ and divide it by $d = 5$ to get

$$\frac{11}{5} = 2\frac{1}{5} \quad \text{or equivalently} \quad \frac{11}{5} = 2 + \frac{1}{5} \quad \text{or equivalently} \quad 11 = 5 \times 2 + 1 .$$

In general, one can divide $n \in N$ by $d \in \mathbb{N}$. Think of as:

$$\begin{array}{r} \text{q} \\ d\sqrt{\ \ n\ } \\ \vdots \\ \hline r \end{array}$$

to get

$$\boxed{\frac{n}{d} = q\frac{r}{d} \quad \text{or equivalently} \quad \frac{n}{d} = q + \frac{r}{d} \quad \text{or equivalently} \quad n = d\,q + r}$$

for some quotient $q \in \mathbb{N} \cup \{0\}$ and some remainder $r \in \mathbb{N} \cup \{0\}$ where $0 \le r < d$.

Theorem. Division Algorithm for $\mathbb{N}$

$$(\ \forall n \in \mathbb{N}\ )\ (\ \forall d \in \mathbb{N}\ )\ (\ \exists\,!\ q \in \mathbb{N} \cup \{0\}\ )\ (\ \exists\,!\ r \in \mathbb{N} \cup \{0\}\ )\ [\ (n = dq + r)\ \wedge\ (0 \le r < d)\ ]$$

equivalently

$$(\ \forall n \in \mathbb{N}\ )\ (\ \forall d \in \mathbb{N}\ )\ (\ \exists\,!\ q \in \mathbb{N} \cup \{0\}\ )\ (\ \exists\,!\ r \in \{0, 1, \ldots, d-1\}\ )\ [\ n = dq + r\ ]$$

Remark. Here: $r \in \{0, 1, 2, \ldots, d-1\}$ so there are d possiblities for r.

Division Algorithm for $\mathbb{Z}$

Theorem. Division Algorithm for $\mathbb{Z}$. (7$^{\text{th}}$ edition, page 62)

$$(\ \forall n \in \mathbb{Z}\ )\ (\ \forall d \in \mathbb{Z} \setminus \{0\}\ )\ (\ \exists\,!\ q \in \mathbb{Z}\ )\ (\ \exists\,!\ r \in \mathbb{Z}\ )\ [\ (n = dq + r)\ \wedge\ (0 \le r < |d|)\ ]$$

equivalently

$$(\ \forall n \in \mathbb{Z}\ )\ (\ \forall d \in \mathbb{Z} \setminus \{0\}\ )\ (\ \exists\,!\ q \in \mathbb{Z}\ )\ (\ \exists\,!\ r \in \{0, 1, \ldots, (|d|-1)\}\ )\ [\ n = dq + r\ ]$$

Remark. Here: $r \in \{0, 1, 2, \ldots, (|d|-1)\}$ so there are d possiblities for r.