$$-2u\cos u + 2\sin u + C = 2(\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}) + C.$$

61. Use integration by parts with

$$u = \arcsin x \quad \text{and} \quad dv = \frac{1}{x^2}\,dx.$$

Then apply Formula 60 of the endpapers, or use the trigonometric substitution $x = \sin u$.

62. Let $x = 3\sec u$ to transform the integral into

$$9\int \sec u \tan^2 u\,du = 9\int (\sec^3 u - \sec u)\,du.$$

Apply Formulas 14 and 28 of the endpapers to obtain

$$\frac{9}{2}(\sec u \tan u - \ln|\sec u + \tan u|) + C_1$$

$$= \frac{9}{2}\left(\frac{1}{9}x(x^2-9)^{1/2} - \ln\left|\frac{x}{3} + \frac{1}{3}(x^2-9)^{1/2}\right|\right) + C_1$$

$$= \frac{1}{2}x(x^2-9)^{1/2} - \frac{9}{2}\ln|x + (x^2-9)^{1/2}| + C.$$

63. Let $x = \sin u$ to transform the integrand into

$$\frac{1}{4}(2\sin u\cos u)^2 = \frac{1}{4}\sin^2(2u) = \frac{1}{8}(1 - \cos 4u).$$

64. Because $2x - x^2 = 1 - (x-1)^2$, let $x = 1 + \sin u$. Then

$$\int x(2x - x^2)^{1/2}\,dx = \int (1 + \sin u)(\cos^2 u)\,du$$

$$= \int \left(\frac{1 + \cos 2u}{2} + \cos^2 u \sin u\right) du$$

$$= \frac{1}{2} u + \frac{1}{2} \sin u \cos u - \frac{1}{3} \cos^3 u + C$$

$$= \frac{1}{2} \sin^{-1}(x - 1) + \frac{1}{2} (x - 1)(2x - x^2)^{1/2}$$

$$- \frac{1}{3} (2x - x^2)^{3/2} + C.$$

The answer can be further simplified to

$$\frac{1}{2} \sin^{-1}(x - 1) + \frac{1}{6} (2x - x^2)^{1/2}(2x^2 - x - 3) + C.$$

65. Write

$$\frac{x - 2}{(2x + 1)^2} = \frac{2x + 1}{2(2x + 1)^2} - \frac{5}{2(2x + 1)^2}.$$

66. Because

$$\frac{2x^2 - 5x - 1}{x^3 - 2x^2 - x + 2} = \frac{1}{x + 1} + \frac{2}{x - 1} - \frac{1}{x - 2},$$

the required antiderivative can be written as

$$\ln\left| \frac{(x + 1)(x - 1)^2}{x - 2} \right| + C.$$

68. Let $u = \sin x$. Then $du = \cos x \, dx$, and we obtain

$$\int \frac{1}{u^2 - 3u + 2} du = \int \frac{1}{(u - 1)(u - 2)} du$$

$$= \int \left(- \frac{1}{u - 1} + \frac{1}{u - 2} \right) du$$

$$= \ln\left| \frac{u - 2}{u - 1} \right| + C = \ln\left(\frac{2 - \sin x}{1 - \sin x} \right) + C.$$

69. The partial fractions decomposition of the integrand is

$$\frac{2}{x + 1} - \frac{3}{(x + 1)^2} + \frac{5}{(x + 1)^4}.$$

70. The substitution $u = \tan x$ yields

$$\int \frac{1}{u^2 + 2x + 2} du = \int \frac{1}{1 + (u + 1)^2} du$$

$$= \tan^{-1}(u + 1) + C = \tan^{-1}(1 + \tan x) + C.$$

71. The partial fractions decomposition of the integrand is

$$\frac{x + 1}{x^2 + 1} + \frac{x}{(x^2 + 1)^2}.$$

72. The substitution $u = \tan \frac{\xi}{2}$ transforms the integral into

$$\int \frac{8 + 4u^2}{(3u^2 + 1)(u^2 + 1)} du.$$

The partial fractions decomposition of the integrand is

$$\frac{10}{3u^2 + 1} - \frac{2}{u^2 + 1},$$

and this leads to the antiderivative

$$\frac{10}{3}\sqrt{3} \tan^{-1}(u\sqrt{3}) - 2 \tan^{-1} u + C$$

$$= \frac{10}{3}\sqrt{3} \tan^{-1}(\sqrt{3} \tan \frac{\xi}{2}) - \xi + C.$$

73. Let $u = x^3 - 1$; the rest is routine.

74. Let $u = \tan\frac{\xi}{2}$. Then the integral becomes

$$\int \frac{1}{u + 2}\,du = \ln|u + 2| + C$$

$$= \ln|2 + \tan\frac{\xi}{2}| + C$$

$$= \ln\left|\frac{2 + 2\cos\xi + \sin\xi}{1 + \cos\xi}\right| + C.$$

76. Let $x = \tan^3 z$. The integral becomes

$$\int \frac{3\tan^2 z\sec^2 z}{\sec^2 z\tan^2 z}\,dz = 3z + C = 3\tan^{-1}(x^{1/3}) + C.$$

77. Use the identity $\sin 2x = 2\sin x\cos x$.

78. Because $\frac{1}{2}(1 + \cos t) = \cos^2(\frac{t}{2})$,

$$\int (1 + \cos t)^{1/2}\,dt = \sqrt{2}\int \left(\frac{1 + \cos t}{2}\right)^{1/2} dt$$

$$= \sqrt{2}\int \cos(\frac{t}{2})\,dt = 2\sqrt{2}\sin(\frac{t}{2}) + C,$$

which may also be written in the form $2\sqrt{1 - \cos t} + C$.

Note: We took the positive square root in the computations above. If this problem had been a definite integral, we'd need to see whether the values of t made $\cos(t/2)$ positive or negative to know which sign to take.

79. Multiply numerator and denominator of the integrand by

$$\sqrt{1 - \sin t}\,.$$

80. Let $u = \tan t$. Then the integral becomes

$$\int \frac{1}{1 - u^2}\,du = \frac{1}{2}\int \left(\frac{1}{1 - u} + \frac{1}{1 + u}\right) du$$

$$= \frac{1}{2}\ln\left|\frac{1 + u}{1 - u}\right| + C = \frac{1}{2}\ln\left|\frac{1 + \tan t}{1 - \tan t}\right| + C.$$

81. Use integration by parts with $u = \ln(x^2 + x + 1)$ and -- if you wish -- $v = x + \frac{1}{2}$ (this will save some trouble later).

82. Let $u = e^x$. The integral becomes

$$I = \int \sin^{-1} u\,du.$$

Now do an integration by parts with $p = \sin^{-1} u$, $dq = du$:

$$I = u\sin^{-1} u - \int u(1 - u^2)^{-1/2}\,du$$

$$= u\sin^{-1} u + (1 - u^2)^{1/2} + C$$

$$= e^x \sin^{-1}(e^x) + (1 - e^{2x})^{1/2} + C.$$

83. Integration by parts, with $u = \arctan x$, $dv = x^{-2}\,dx$. The new integrand has the partial fractions decomposition

$$\frac{1}{x} - \frac{x}{x^2 + 1}\,.$$