

$$-2u \cos u + 2 \sin u + C = 2(\sin \sqrt{x} - \sqrt{x} \cos \sqrt{x}) + C.$$

61. Use integration by parts with

$$u = \arcsin x \quad \text{and} \quad dv = \frac{1}{x^2} dx.$$

Then apply Formula 60 of the endpapers, or use the trigonometric substitution $x = \sin u$.

62. Let $x = 3 \sec u$ to transform the integral into

$$9 \int \sec u \tan^2 u \, du = 9 \int (\sec^3 u - \sec u) \, du.$$

Apply Formulas 14 and 28 of the endpapers to obtain

$$\begin{aligned} & \frac{9}{2} (\sec u \tan u - \ln |\sec u + \tan u|) + C_1 \\ &= \frac{9}{2} \left(\frac{1}{9} x(x^2 - 9)^{1/2} - \ln \left| \frac{x}{3} + \frac{1}{3} (x^2 - 9)^{1/2} \right| \right) + C_1 \\ &= \frac{1}{2} x(x^2 - 9)^{1/2} - \frac{9}{2} \ln |x + (x^2 - 9)^{1/2}| + C. \end{aligned}$$

63. Let $x = \sin u$ to transform the integrand into

$$\frac{1}{4} (2 \sin u \cos u)^2 = \frac{1}{4} \sin^2(2u) = \frac{1}{8} (1 - \cos 4u).$$

64. Because $2x - x^2 = 1 - (x - 1)^2$, let $x = 1 + \sin u$. Then

$$\begin{aligned} \int x(2x - x^2)^{1/2} dx &= \int (1 + \sin u)(\cos^2 u) \, du \\ &= \int \left(\frac{1 + \cos 2u}{2} + \cos^2 u \sin u \right) du \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2}u + \frac{1}{2}\sin u \cos u - \frac{1}{3}\cos^3 u + C \\
&= \frac{1}{2}\sin^{-1}(x-1) + \frac{1}{2}(x-1)(2x-x^2)^{1/2} \\
&\quad - \frac{1}{3}(2x-x^2)^{3/2} + C.
\end{aligned}$$

The answer can be further simplified to

$$\frac{1}{2}\sin^{-1}(x-1) + \frac{1}{6}(2x-x^2)^{1/2}(2x^2-x-3) + C.$$

65. Write

$$\frac{x-2}{(2x+1)^2} = \frac{2x+1}{2(2x+1)^2} - \frac{5}{2(2x+1)^2}.$$

66. Because

$$\frac{2x^2-5x-1}{x^3-2x^2-x+2} = \frac{1}{x+1} + \frac{2}{x-1} - \frac{1}{x-2},$$

the required antiderivative can be written as

$$\ln \left| \frac{(x+1)(x-1)^2}{x-2} \right| + C.$$

68. Let $u = \sin x$. Then $du = \cos x dx$, and we obtain

$$\begin{aligned}
\int \frac{1}{u^2-3u+2} du &= \int \frac{1}{(u-1)(u-2)} du \\
&= \int \left(-\frac{1}{u-1} + \frac{1}{u-2} \right) du \\
&= \ln \left| \frac{u-2}{u-1} \right| + C = \ln \left(\frac{2-\sin x}{1-\sin x} \right) + C.
\end{aligned}$$

69. The partial fractions decomposition of the integrand is

$$\frac{2}{x+1} - \frac{3}{(x+1)^2} + \frac{5}{(x+1)^4}.$$

70. The substitution $u = \tan x$ yields

$$\begin{aligned} \int \frac{1}{u^2 + 2x + 2} du &= \int \frac{1}{1 + (u+1)^2} du \\ &= \tan^{-1}(u+1) + C = \tan^{-1}(1 + \tan x) + C. \end{aligned}$$

71. The partial fractions decomposition of the integrand is

$$\frac{x+1}{x^2+1} + \frac{x}{(x^2+1)^2}.$$

72. The substitution $u = \tan \frac{\xi}{2}$ transforms the integral into

$$\int \frac{8 + 4u^2}{(3u^2 + 1)(u^2 + 1)} du.$$

The partial fractions decomposition of the integrand is

$$\frac{10}{3u^2 + 1} - \frac{2}{u^2 + 1},$$

and this leads to the antiderivative

$$\begin{aligned} &\frac{10}{3} \sqrt{3} \tan^{-1}(u\sqrt{3}) - 2 \tan^{-1}u + C \\ &= \frac{10}{3} \sqrt{3} \tan^{-1}(\sqrt{3} \tan \frac{\xi}{2}) - \xi + C. \end{aligned}$$

73. Let $u = x^3 - 1$; the rest is routine.

74. Let $u = \tan \frac{\xi}{2}$. Then the integral becomes

$$\begin{aligned}\int \frac{1}{u+2} du &= \ln|u+2| + C \\ &= \ln\left|2 + \tan \frac{\xi}{2}\right| + C \\ &= \ln\left|\frac{2 + 2\cos\xi + \sin\xi}{1 + \cos\xi}\right| + C.\end{aligned}$$

76. Let $x = \tan^3 z$. The integral becomes

$$\int \frac{3 \tan^2 z \sec^2 z}{\sec^2 z \tan^2 z} dz = 3z + C = 3 \tan^{-1}(x^{1/3}) + C.$$

77. Use the identity $\sin 2x = 2 \sin x \cos x$.

78. Because $\frac{1}{2}(1 + \cos t) = \cos^2\left(\frac{t}{2}\right)$,

$$\begin{aligned}\int (1 + \cos t)^{1/2} dt &= \sqrt{2} \int \left(\frac{1 + \cos t}{2}\right)^{1/2} dt \\ &= \sqrt{2} \int \cos\left(\frac{t}{2}\right) dt = 2\sqrt{2} \sin\left(\frac{t}{2}\right) + C,\end{aligned}$$

which may also be written in the form $2\sqrt{1 - \cos t} + C$.

Note: We took the positive square root in the computations above. If this problem had been a definite integral, we'd need to see whether the values of t made $\cos(t/2)$ positive or negative to know which sign to take.

79. Multiply numerator and denominator of the integrand by

$$\sqrt{1 - \sin t}.$$

80. Let $u = \tan t$. Then the integral becomes

$$\begin{aligned} \int \frac{1}{1 - u^2} du &= \frac{1}{2} \int \left(\frac{1}{1 - u} + \frac{1}{1 + u} \right) du \\ &= \frac{1}{2} \ln \left| \frac{1 + u}{1 - u} \right| + C = \frac{1}{2} \ln \left| \frac{1 + \tan t}{1 - \tan t} \right| + C. \end{aligned}$$

81. Use integration by parts with $u = \ln(x^2 + x + 1)$ and -- if you wish -- $v = x + \frac{1}{2}$ (this will save some trouble later).

82. Let $u = e^x$. The integral becomes

$$I = \int \sin^{-1} u \, du.$$

Now do an integration by parts with $p = \sin^{-1} u$, $dq = du$:

$$\begin{aligned} I &= u \sin^{-1} u - \int u(1 - u^2)^{-1/2} du \\ &= u \sin^{-1} u + (1 - u^2)^{1/2} + C \\ &= e^x \sin^{-1}(e^x) + (1 - e^{2x})^{1/2} + C. \end{aligned}$$

83. Integration by parts, with $u = \arctan x$, $dv = x^{-2} dx$. The new integrand has the partial fractions decomposition

$$\frac{1}{x} - \frac{x}{x^2 + 1}.$$