36. By parts: Let $u = \tan^{-1}x$, $dv = x^2\,dx$. You'll get

$$\frac{1}{3}x^3\tan^{-1}x - \frac{1}{3}\int\left(x - \frac{x}{1+x^2}\right)dx$$

$$= \frac{1}{3}x^3\tan^{-1}x - \frac{1}{6}x^2 + \frac{1}{6}\ln(1+x^2) + C.$$

37. Suggestion: Develop a reduction formula for

$$\int x(\ln x)^n\,dx$$

by parts; take $u = (\ln x)^n$ and $dv = x\,dx$. Then apply your formula iteratively to evaluate the given antiderivative. You should find that

$$\int x(\ln x)^n\,dx = \frac{1}{2}x^2(\ln x)^n - \frac{n}{2}\int x(\ln x)^{n-1}\,dx.$$

38. Let $x = \tan\xi$ to transform the integral into

$$\int \csc\xi\,d\xi = \ln|\csc\xi - \cot\xi| + C$$

$$= \ln\frac{(x^2+1)^{1/2} - 1}{x} + C.$$

39. Let $u = e^x$, then $u = \tan z$ to obtain $\int \sec^3 z\,dz$.

40. Note that $4x - x^2 = 4 - (x-2)^2$; let $x = 2 + 2\sin u$. This substitution transforms the integral into

$$\int 2(1 + \sin u)\, du = 2(u - \cos u) + C$$

$$= 2 \arcsin \frac{x - 2}{2} - (4x - x^2)^{1/2} + C.$$

41. Let $x = 3 \sec \xi$.

42. Let $u = 7x + 1$; the answer is $-\dfrac{112x + 1}{11760(7x + 1)^{16}} + C.$

43. Use the method of partial fractions.

44. Divide denominator into numerator, then use the method of partial fractions as in the solution to Problem 32 on page 596. The antiderivative is

$$4x - \frac{2}{3} \ln|x + 1| + \frac{1}{3} \ln|x^2 - x + 1|$$

$$- \frac{4}{3}\sqrt{3} \tan^{-1}\left(\frac{1}{3}\sqrt{3}\,\{2x - 1\}\right) + C.$$

45. Write the integrand as $\sec^3 x - \sec x$, then apply the formulas of the endpapers.

46. Here is a quick way to obtain the partial fractions decomposition that you need:

$$\frac{x^2 + 2x + 2}{(x + 1)^3} = \frac{(x + 1)^2 + 1}{(x + 1)^3}.$$

47. The partial fractions decomposition of the integrand is

$$\frac{1}{x + 1} + \frac{2}{x^4}.$$

48. The partial fractions decomposition of the integrand is

$$-\frac{1}{x^2 + 1} + \frac{8}{4x + 1},$$

and the antiderivative is therefore

$$2 \ln|4x + 1| - \tan^{-1}x + C.$$

49. The partial fractions decomposition of the integrand has the form

$$\frac{A}{x - 1} + \frac{Bx + C}{x^2 + x + 1} + \frac{D}{(x - 1)^2} + \frac{Ex + F}{(x^2 + x + 1)^2}.$$

The simultaneous equations are

$$\begin{aligned}
A + B &= 3,\\
A - B + C + D &= -1,\\
A - C + 2D + E &= 2,\\
-A - B + 3D - 2E + F &= -12,\\
-A + B - C + 2D + E - 2F &= -2,\\
-A + C + D + F &= 1.
\end{aligned}$$

Their solution: a = 1, B = 2, C = 1, D = -1, E = 4, F = 2. None of the antiderivatives is difficult, and the final answer may be written in the form

$$\ln|x - 1| + \ln(x^2 + x + 1) + \frac{1}{x - 1} - \frac{2}{x^2 + x + 1} + C.$$

50. First, $x^4 + 4x^2 + 8 = (x^2 + 2)^2 + 4$. So

$$\int \frac{x}{x^4 + 4x^2 + 8}\,dx = \int \frac{x}{(x^2 + 2)^2 + 4}\,dx$$

$$= \frac{1}{4} \int \frac{x}{\{(x^2 + 2)/2\}^2 + 1}\,dx$$

$$= \frac{1}{4} \arctan\left(\frac{x^2 + 2}{2}\right) + C.$$

51. Use the substitution $u = \tan\frac{\xi}{2}$.

52. Let $x = u^3$ to transform the given integral into

$$3\int u(1 + u^2)^{3/2}\, du = \frac{3}{5}(1 + u^2)^{5/2} + C$$

$$= \frac{3}{5}(1 + x^{2/3})^{5/2} + C.$$

54. Let $x = u^6$. The integral becomes

$$6\int \frac{1}{u^4(1 + u^2)}\, du.$$

Next let $u = \tan z$. The integral above is transformed into

$$6\int \frac{\sec^2 z}{\tan^4 z \sec^2 z}\, dz = 6\int \cot^4 z\, dz$$

$$= 6\int (\csc^2 z - 1)\cot^2 z\, dz$$

$$= 6\int \{\cot^2 z \csc^2 z - (\csc^2 z - 1)\}\, dz$$

$$= 6(-\frac{1}{3}\cot^3 z + \cot z + z) + C$$

$$= 6(\frac{1}{u} + \arctan u - \frac{1}{3u^3}) + C$$

$$= 6x^{-1/6} + 6\tan^{-1}(x^{1/6}) - 2x^{-1/2} + C.$$

55. $(\tan z)(\sec^2 z - 1) = (\sec z)(\sec z \tan z) - \tan z.$

56. Use Formula 39 of the endpapers; alternatively, repeated use of the half-angle formulas and other trigonometric identities may be used to transform the integral into

$$\frac{1}{8}\int \{1 + \cos 2w - \frac{1}{2}(1 + \cos 4w) - (\cos 2w)(1 - \sin^2 2w)\}\, dw$$

$$= \frac{1}{8}\left(\frac{1}{2}w - \frac{1}{4}\sin 2w \cos 2w + \frac{1}{6}\sin^3 2w\right) + C.$$

57. Because $e^{2x^2} = {e^{x^2}}^2$, let $u = e^{x^2}$. Then the pattern will be come clear.

58. Note that

$$\frac{\cos^3 x}{(\sin x)^{1/2}} = \frac{(1 - \sin^2 x)(\cos x)}{(\sin x)^{1/2}}$$

$$= (\sin x)^{-1/2}\cos x - (\sin x)^{3/2}\cos x.$$

So the antiderivative is

$$\frac{2}{5}(\sin x)^{1/2}(5 - \sin^2 x) + C.$$

59. Use integration by parts; choose as dv the "most difficult part of the integrand that one can actually integrate:"

$$u = x^2, \quad dv = xe^{-x^2}\,dx.$$

60. Let $u = \sqrt{x}$. The integral becomes

$$\int 2u \sin u \, du.$$

Now use integration by parts with $p = 2u$, $dq = \sin u \, du$. The resulting antiderivative is