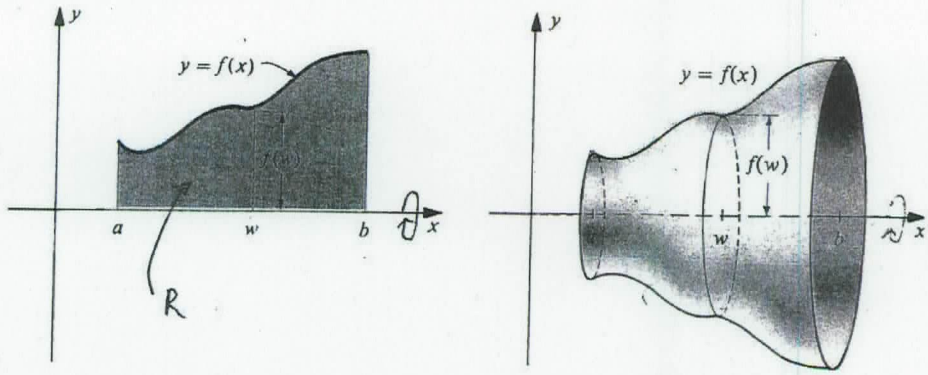
§ 6.2 & 6.3 Volume of Solids of Revolution

(i) Start out with a region R in the xy-plane

(ii) Revolve (i.e. spin) the region R around a horizontal or vertical line to form your "Solid of Revolution"



(i) (ii)

(iii) Find the volume V of this Solid of Revolution by expressing V as a definite integral and integrating it.

Game Plan

① partition the appropriate axis (either the x-axis or y-axis)

If you partition the z-axis, then $V = \int_{\#}^{\#} (\text{some function of } z)\, dz$.

see next page (ii) (iii)

② form Riemann typical rectangles as if you were looking for the Area of R.

③ revolve the typical rectangles to get "typical elements", here an element will be: disk or washer or shell.

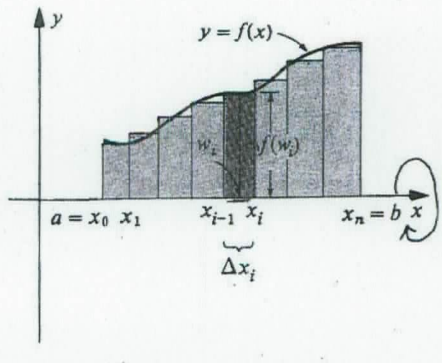
④ Find the volume $\tilde{V}$ of a typical element. $\tilde{V}$ will look like $\tilde{V} = (\text{some function of } z)\, \Delta z$.

⑤ Sum the volume of all the typical elements resulting from your partition

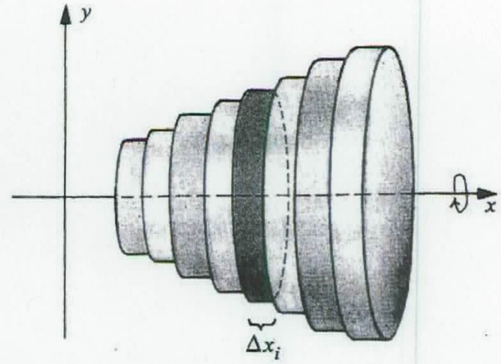
⑥ take the limit as $\Delta z \to 0$ to get

$$V = \int_{\#}^{\#} (\text{that some function of } z)\, \Delta z.$$

Disk Method revolve abt x-axis & partition x-axis



(i)



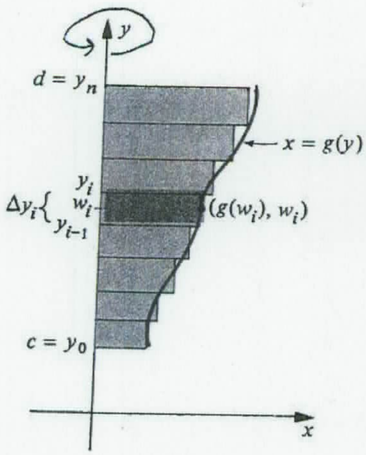
(ii)

no hole

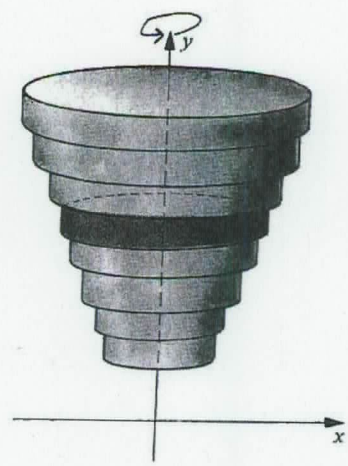
⇓

disk

Disk Method revolve abt y-axis & partition y-axis



(i)



(ii)

no hole

⇓

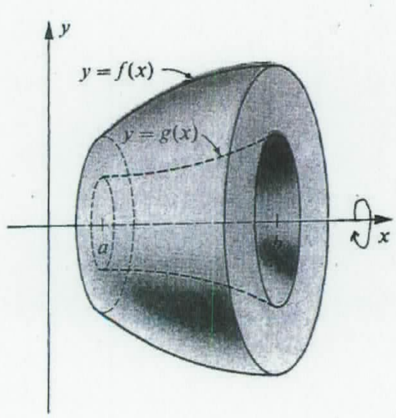
disk

Washer Method revolve abt x-axis
partitioned x-axis

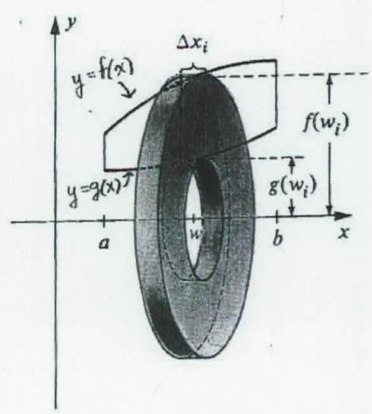
has hole

⇓

washer



(i)



(ii)

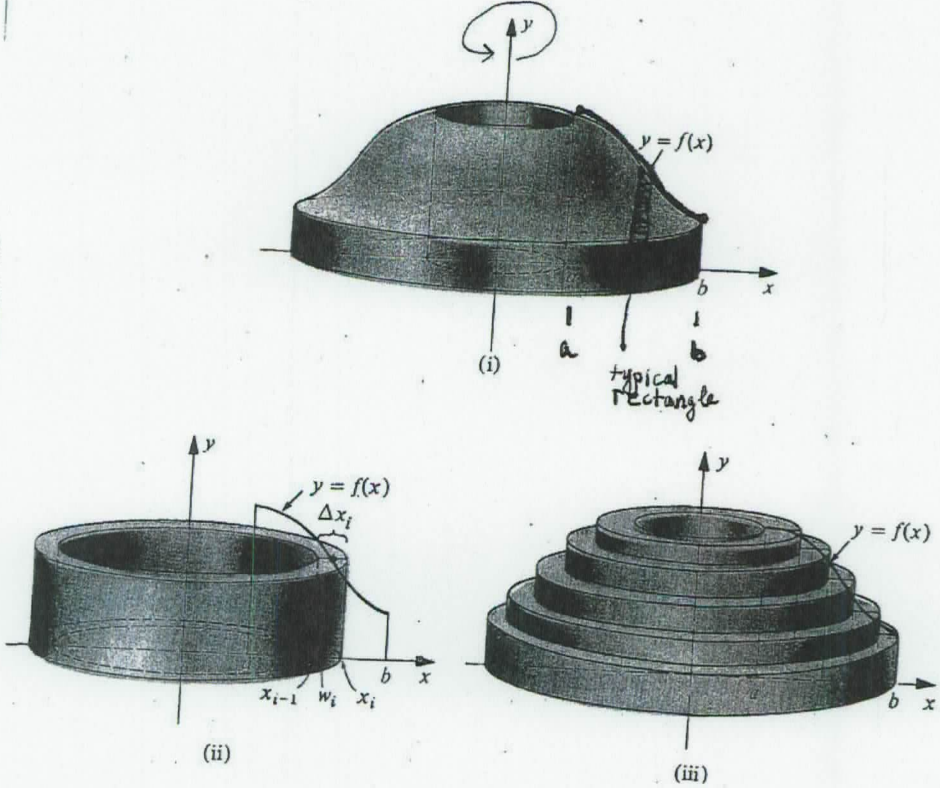
Shell Method

Picture
- our textbook pages 433 - 434

(or)

- Swoko p 290 (below)

Revolving about y-axis → partition x-axis



- Let's say partitioned z-axis (where $z = x$ or $z = y$)
- So $V = \int_{z=?}^{z=?} (\text{some function of } z)\, dz$

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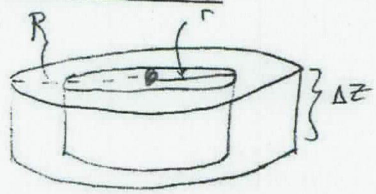
§6.2 **Disk/Washer Method : partition (∥-to) axis of revolution**

- typical element = disk or washer
- Disk (no hole)

$$\text{Volume} = (\text{area of base})\ (\text{height})$$
$$= (\pi r^2)\ (\Delta z)$$

- Washer (hole)



$$\text{Volume} = (\text{Volume of big}) - (\text{Volume of little})$$
$$= \pi R^2 \Delta z - \pi r^2 \Delta z$$
$$= \pi (R^2 - r^2) \Delta z$$
$$\neq \pi (R-r)^2 \Delta z$$

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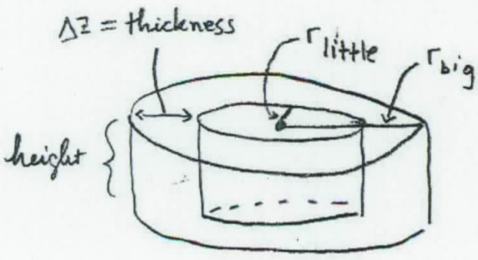
§6.3 **Shell Method : partition ⊥ to axis of revolution**

- typical element = shell.



Volume of typical shell

$$= 2\pi\,(\text{average radius})\,(\text{height})\,(\text{thickness})$$
$$= 2\pi \left(\frac{r_{big} + r_{little}}{2}\right)(\text{height})\underbrace{(r_{big} - r_{little})}_{(\Delta z)}$$
$$= 2\pi\,((\text{avg})\text{ radius})\,(\text{height})\,(\Delta z)$$

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