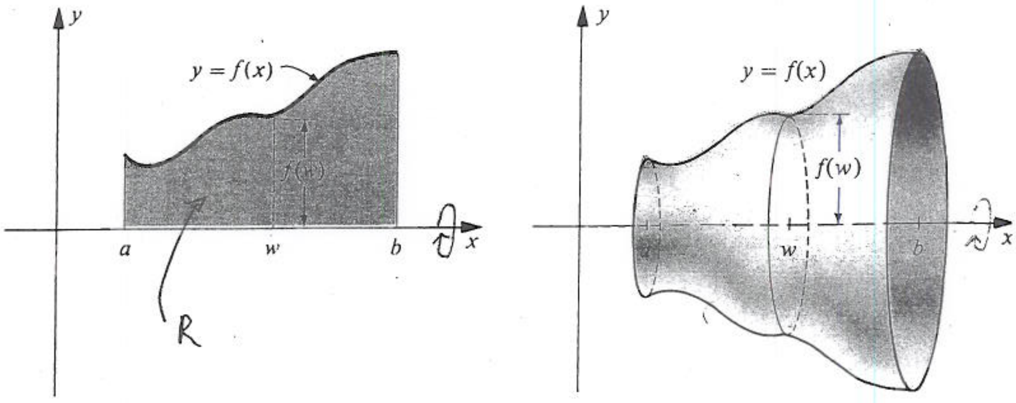
§ 7.2, 7.3 Volume of Solids of Revolution

(i) Start out with a region R in the xy-plane

(ii) Revolve (i.e. spin) the region R around a horizontal or vertical line to form your "Solid of Revolution"



(i) (ii)

(iii) Find the volume V of this Solid of Revolution by expressing V as a definite integral and integrating it.

Game Plan

① partition the appropriate axis (either the x-axis or y-axis)

If you partition the z-axis, then $V = \int_{\#}^{\#} (\text{some function of } z)\, dz$.

see (iii) next page

(ii) → ② form Riemann typical rectangles as if you were looking for the Area of R.

(iii) → ③ revolve the typical rectangles to get "typical elements".

here an element will be: disk or washer or shell.

④ Find the volume $\tilde{V}$ of a typical element. $\tilde{V}$ will look like

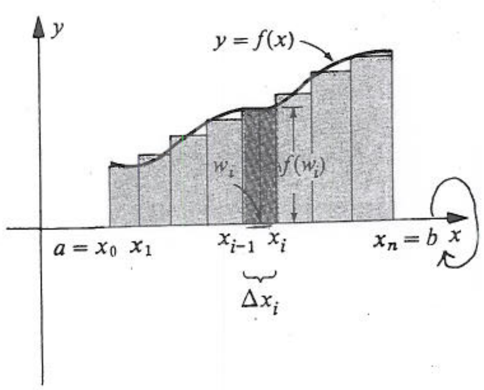
$\tilde{V} = (\text{some function of } z)\, \Delta z$.

⑤ Sum the volume of all the typical elements resulting from your partition

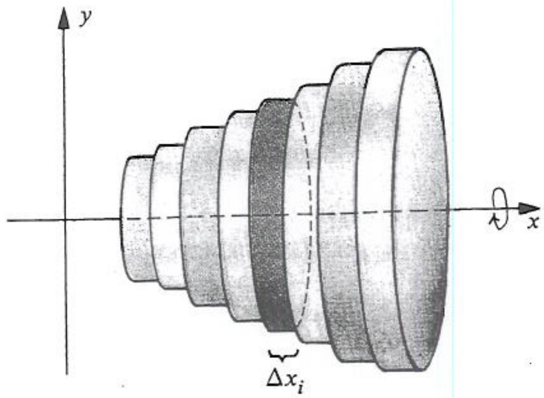
⑥ take the limit as $\Delta z \to 0$ to get

$V = \int_{\#}^{\#} (\text{that some function of } z)\, \Delta z.$

Disk Method revolve abt x-axis & partition x-axis



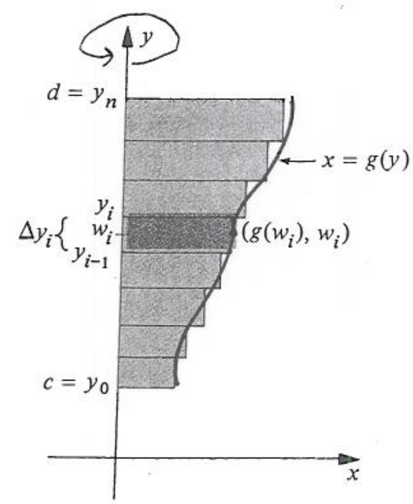
(i)



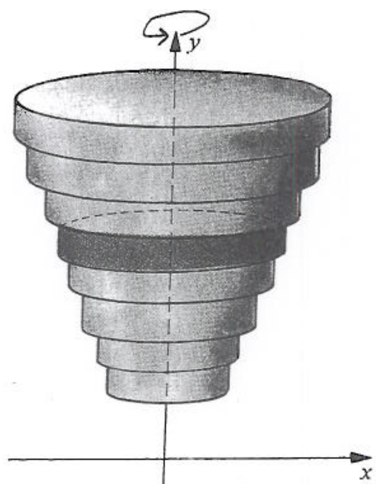
(ii)

no hole
⇓
disk

Disk Method revolve abt y-axis & partition y-axis



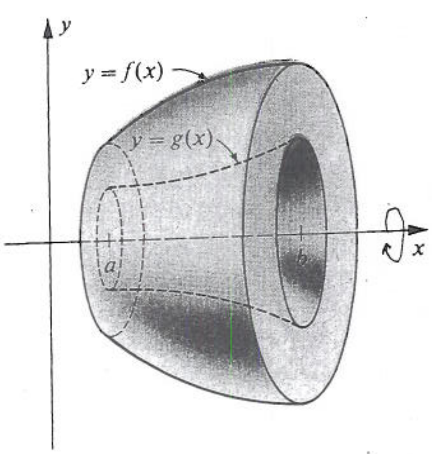
(i)



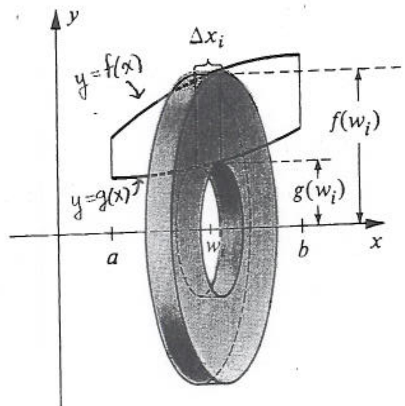
(ii)

no hole
⇓
disk

Washer Method revolve abt x-axis partitioned x-axis



(i)



(ii)

has hole
⇓
washer

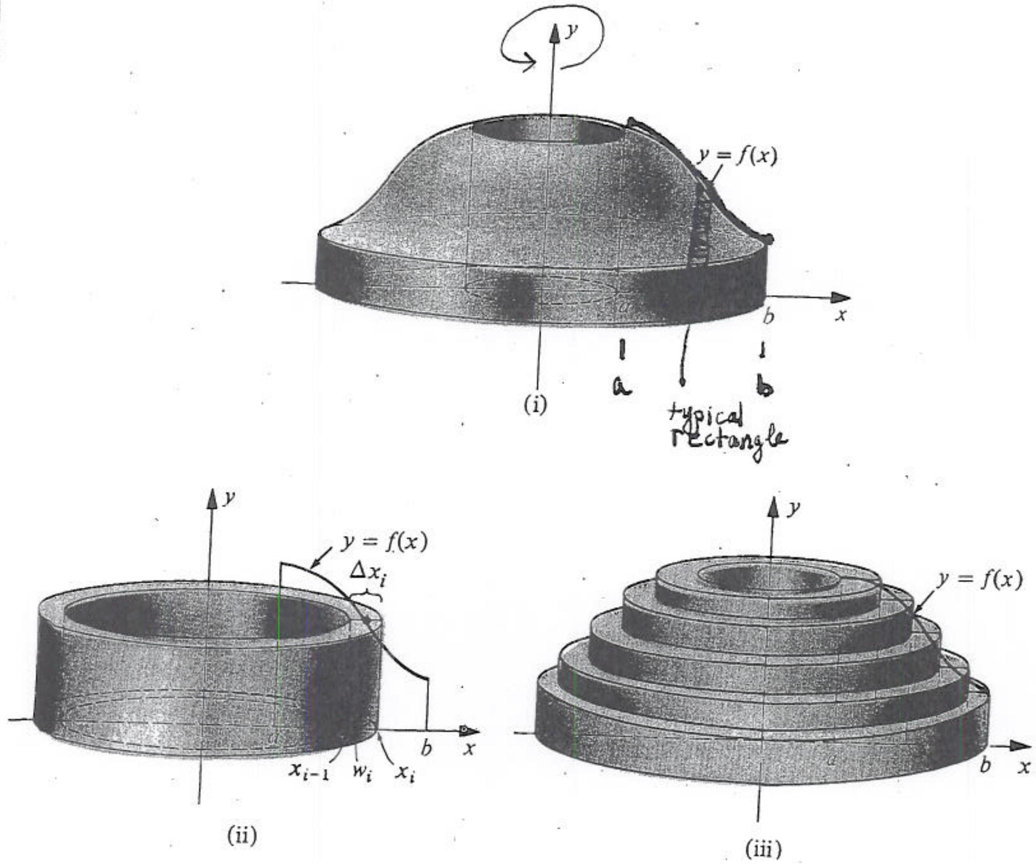
Shell Method

Picture • our textbook Figure 7.3.3 (p. 460)

(or)

• Swoko p 290 (below)

Revolving about y-axis → partition x-axis



• Let's say partitioned z-axis (where $z = x$ or $z = y$)

• So $V = \int_{z=?}^{z=?}$ (some function of z) dz

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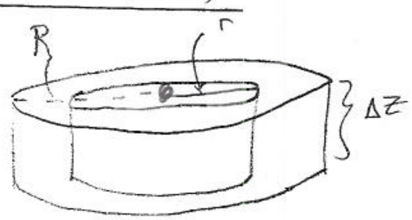
§7.2 **Disk/Washer Method : partition (∥-to) axis of revolution**

• typical element = disk or washer

• Disk (no hole)

Volume = (area of base) (height)
$= (\pi r^2) (\Delta z)$

• Washer (hole)



Volume = (Volume of big) − (Volume of little)
$= \pi R^2 \Delta z - \pi r^2 \Delta z$
$= \pi (R^2 - r^2) \Delta z$
$\neq \pi (R - r)^2 \Delta z$

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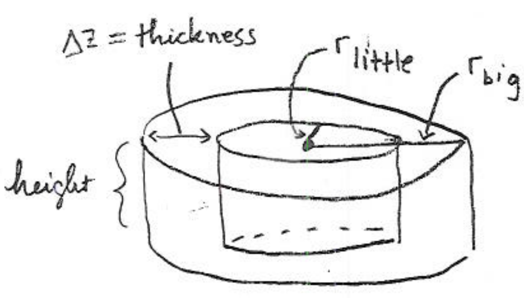
§7.3 **Shell Method : partition ⊥ to axis of revolution**

• typical element = Shell.



Volume of typical shell
= 2π (average radius) (height) (thickness)
$= 2\pi \left(\frac{r_{big} + r_{little}}{2}\right)(\text{height})(r_{big} - r_{little})$
= 2π ((avg) radius) (height) (ΔZ)