Polar Coordinate

Conversion

A point P in the plane with:
Cartesian coordinates (x, y) and polar coordinates (r, θ) satisfies the following.

$$x = r\cos\theta \qquad\qquad r^2 = x^2 + y^2$$

$$y = r\sin\theta \qquad\qquad \tan\theta = \frac{y}{x}\ .$$

Graphing

The period of $y = \sin(k\theta)$ and of $y = \cos(k\theta)$ is $\frac{2\pi}{k}$.
To sketch graphs, divide the period by 4.

Area

Let $A(r, \theta)$ be the area of a sector of a circle with cental angle θ radians and radius r.
Comparing $A(r, \theta)$ to the area of the whole circle lead us to the proportion:

$$\frac{A(r,\theta)}{A(r,2\pi)} = \frac{\theta}{2\pi} \quad\Rightarrow\quad \frac{A(r,\theta)}{\pi r^2} = \frac{\theta}{2\pi} \quad\Rightarrow\quad A(r,\theta) = \frac{\theta}{2\pi}\,\frac{\pi r^2}{1}$$

from which we concluded that

$$A(r,\theta) = \frac{\theta r^2}{2}\ .$$

Now consider a function $r = f(\theta)$ which determines a curve in the plane where

(1) $f\colon [\alpha, \beta] \ \rightarrow\ [0, \infty]$
(2) f is continuous on $[\alpha, \beta]$
(3) $\beta - \alpha \leq 2\pi$.

Then the area bounded by $r = f(\theta)$ and $\theta = \alpha$ and $\theta = \beta$ is

$$A = \frac{1}{2}\int_{\theta=\alpha}^{\theta=\beta} [f(\theta)]^2\, d\theta\ .$$