MARK BOX		
PROBLEM	POINTS	
1 a–d	20	
2	5	
3	5	
bonus	2	
Total	30	
%	100	

Math 142.501 Prof. Girardi

Fall 98 Exam 4 11/24/98

NAME: Key

INSTRUCTIONS:

1. To receive credit you must:
   a. WORK IN A LOGICAL FASHION,
      SHOW ALL YOUR WORK,
      INDICATE YOUR REASONING.
   b. when applicable put your answer on/in the line/box provided
   c. if no such line/box is provided, then box your answer
   d. if you use your calculator, give an explanation of what you did on it.
2. The MARK BOX indicates the problems along with their points.
   Check that your copy of the exam has all of the problems.
3. During this test, do not leave your seat.
   If you have a question, raise your hand.
   When you finish: turn your exam over, put your pencil down, raise your hand.
4. This closed book/notes quiz covers (from *Calculus*, by Varberg and Purcell) : § 11.2 – 11.5 .

Problem Source:

1. Look at problems: § 11.9 # 19 & 25 and § 11.4 # 15 & 42a (with $(-1)^n$ added for fun)
2. Look at problem § 11.2 # 20
3. Look at problem § 11.5 # 32

bonus. just for fun

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1. Determine whether each of the following 4 series is absolutely convergent, conditionally convergent, or divergent. CLEARLY explain your reasoning and indicate the test(s) used. No credit will be given for work that does not make sense to us!

1a. $\displaystyle\sum_{n=7}^{\infty}(-1)^n\frac{n}{1+n^2}$

_____ absolutely convergent

__✓__ conditionally convergent

_____ divergent

① First consider $\sum \left| (-1)^n \frac{n}{1+n^2} \right| = \sum \frac{n}{1+n^2}$

L.C.T. $\frac{n}{1+n^2} \sim \frac{n}{n^2} = \frac{1}{n} = b_n$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n}{1+n^2}\cdot\frac{n}{1} = \lim_{n\to\infty}\frac{n^2}{1+n^2} = \lim_{n\to\infty}\frac{1}{\frac{1}{n^2}+1} = 1$ $0 < 1 < \infty$

LCT $\Rightarrow$ $\sum \frac{1}{n}$ & $\sum \frac{n}{1+n^2}$ do the same thing.

$\sum \frac{1}{n}$ diverges p-series $p = 1 \leq 1$ } so $\sum \frac{n}{1+n^2}$ diverges.

② A.S.T. $\lim_{n\to\infty} \frac{n}{1+n^2} = \lim_{n\to\infty} \frac{\frac{1}{n}}{\frac{1}{n^2}+1} = \frac{0}{0+1} = 0.$

A.S.T. $\Rightarrow$ $\sum (-1)^n \frac{n}{1+n^2}$ converges

1b. $\sum_{n=7}^{\infty}(-1)^n \frac{n^2}{n!}$

$\checkmark$ absolutely convergent
_____ conditionally convergent
_____ divergent

First consider $\sum \left| (-1)^n \frac{n^2}{n!} \right| = \sum \frac{n^2}{n!}$

That $n!$ suggests Ratio Test.

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^2}{(n+1)!} \cdot \frac{n!}{n^2}$$

$$= \lim_{n\to\infty} \left(\frac{n+1}{n}\right)^2 \cdot \frac{n!}{n!\,(n+1)}$$

$$= \lim_{n\to\infty} \left(\frac{1+\frac{1}{n}}{1}\right)^2 \cdot \frac{1}{n+1}$$

$$= \frac{1+0}{1} \cdot 0 = 0 < 1$$

1c. $\sum_{n=7}^{\infty}\left(\frac{-1}{\ln n}\right)^n$

✓ absolutely convergent
_____ conditionally convergent
_____ divergent

First consider $\sum \left| \left(\frac{-1}{\ln n}\right)^n \right| = \sum \left(\frac{1}{\ln n}\right)^n$.

That n^{th} power suggests Root Test

$$\lim_{n\to\infty} a_n^{1/n} = \lim_{n\to\infty}\left[\left(\frac{1}{\ln n}\right)^n\right]^{1/n}$$

$$= \lim_{n\to\infty}\frac{1}{\ln n} = 0 < 1.$$

1d. $\displaystyle\sum_{n=7}^{\infty}(-1)^n\frac{n+1}{10n+12}$

_____ absolutely convergent
_____ conditionally convergent
✓ divergent

$$\lim_{n\to\infty} \frac{n+1}{10n+12} = \lim_{n\to\infty} \frac{1+\frac{1}{n}}{10+\frac{12}{n}} = \frac{1}{10}.$$

$$\Rightarrow \lim_{n\to\infty} (-1)^n \frac{n+1}{10n+12} \quad \text{DNE}$$ ∵ it is oscillating btw $\approx -\frac{1}{10}$ & $\frac{1}{10}$.

$N^{\underline{th}}$ - term test for divergence

2. Express the number

$$0.3671717171\ldots\ldots$$

as an infinite series and as a ratio of two integers. Please box your answers.

$$.367171\,71\,71\ldots = \frac{36}{(10)^2} + \frac{71}{(10)^4} + \frac{71}{(10)^6} + \frac{71}{(10)^8} + \cdots$$

$$= \frac{36}{100} + 71 \cdot \sum_{n=2}^{\infty} \frac{1}{10^{2n}} \quad \text{so} \quad 10^{2n} = (10^2)^n = 100^n$$

$$= \frac{36}{100} + 71 \sum_{n=2}^{\infty} \frac{1}{100^n}$$

$$= \frac{36}{100} + 71 \cdot \left(\frac{1}{100}\right)^2 \sum_{n=2}^{\infty} \left(\frac{1}{100}\right)^n \left(\frac{1}{100}\right)^{-2}$$

$$= \frac{36}{100} + \frac{71}{100^2} \sum_{n=2}^{\infty} \left(\frac{1}{100}\right)^{n-2}$$

$$= \frac{36}{100} + \frac{71}{(100)^2} \sum_{n=0}^{\infty} \left(\frac{1}{100}\right)^n$$

$$= \frac{36}{100} + \frac{71}{(100)^2} \left[\frac{1}{1 - 1/100}\right]$$

$$= \frac{36}{100} + \frac{71}{(100)^2} \cdot \frac{100}{99}$$

$$= \frac{36 \cdot 99}{100 \cdot 99} + \frac{71}{100 \cdot 99} = \frac{3564 + 71}{9900}$$

$$= \boxed{\frac{3635}{9900}}$$

3. Give an example of two series $\sum a_n$ and $\sum b_n$, both convergent, such that $\sum a_n b_n$ diverges. Clearly explain your reasoning.

There are lots of such example.

One way to generate them is to take

$$a_n = \frac{(-1)^n}{n^p} \quad \& \quad b_n = \frac{(-1)^n}{n^q} \quad \text{so} \quad a_n b_n = \frac{1}{n^{p+q}}$$

with $0 < p$, $0 < q$, and $0 < p+q < 1$

A.S.T. $\Rightarrow$ $\sum a_n$ & $\sum b_n$ conv.

$\tilde{p}$-series w/ "$\tilde{p} = p+q$" $\Rightarrow$ $\sum a_n b_n$ diverges.

BONUS. Make up a CHALLENGING series and then determine if it is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=7}^{\infty}$$

_____ absolutely convergent
_____ conditionally convergent
_____ divergent