MARK BOX		
Problem	Points	
1	30	
2	15	
3	20	
4	20	
5	15	
Total	100	

MATH 142 sections 004 & 005
FALL 1993 EXAM # 3

NAME: ____________

SSN: ____________

Instructions:

(1) To receive credit, you must work in a logical fashion, SHOW ALL YOUR WORK, INDICATE YOUR REASONING, and when applicable put your answer on the line (or in the box) provided.
(2) The "Mark Box" indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) During this test, do not leave your seat. Raise your hand if you have a question. When you finish, turn your exam over, put your pencil down, and raise your hand.
(4) "Formula sheets" are not allowed. Calculators are allowed.
(5) This is a closed book/closed notes exam covering (from *Calculus* by Edwards & Penny) sections 12.7–12.9, 10.1–10.2.

1. Find the interval of convergence for each of the below power series. Do not forget to "check the endpoints." Has parts a), b), and c).

a) $\sum_{n=1}^{\infty} \frac{x^n}{n!}$ has interval of convergence $(-\infty, \infty)$.

$\rho = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n}\right| = |x| \cdot \lim_{n\to\infty} \frac{1}{n+1} = |x| \cdot 0 < 1.$

pg 584 b) $\sum_{n=1}^{\infty} \frac{(2n)!\, x^n}{n!}$ has interval of convergence $[0,0]$ or $\{0\}$.

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(2n+2)!}{(2n)!} \frac{x^{n+1}}{x^n} \frac{n!}{(n+1)!}\right| = |x| \lim_{n\to\infty} \frac{(2n+1)(2n+2)}{n+1} =$

$= |x| \lim_{n\to\infty} \frac{4n^2+6n+2}{n+1} = |x| \lim_{n\to\infty} \frac{4+\frac{6}{n}+\frac{2}{n^2}}{\frac{1}{n}+\frac{1}{n^2}} = |x| \cdot \infty$

pg 584 c) $\sum_{n=1}^{\infty} \frac{(2x-6)^n}{10n+17}$ has interval of convergence $\left[\frac{5}{2}, \frac{7}{2}\right)$.

$\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(2x-6)^{n+1}}{(2x-6)^n} \frac{10n+17}{10(n+1)+17}\right| = |2x-6| \lim_{n\to\infty} \frac{10n+17}{10n+27} = 2|x-3| \lim_{n\to\infty} \frac{10+\frac{17}{n}}{10+\frac{27}{n}} = 2|x-3|$

Conv when $2|x-3| < 1 \iff |x-3| < \frac{1}{2}$ $\quad 2\tfrac{1}{2} \quad 3 \quad 3\tfrac{1}{2}$

$x=\frac{7}{2}$: $\sum \frac{1}{10n+17}$ divg (harmonic series) ||| $x=2\frac{1}{2}=\frac{5}{2}$: $\sum \frac{(-1)^n}{10n+17}$ conv. Alt. Ser. Test

2. For each of the following functions, write a power series expansion (namely the Taylor series) about the point $a=0$. Write your answer in closed form (i. e. using the $\sum$ sign). Indicate the values of x for which the expansion is valid.

p 575 a) $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ valid for all x

p 575 b) $\cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$ valid for all x

p 575 c) $\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$ valid for all x

Geometric Series d) $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$ valid for $|x|<1$
p 575

3. Find a power series expansion (namely the Taylor series) of the below functions about the point a. Be clever and use your answers to question 2. Write your answer in closed form. Indicate the values of x for which the expansion is valid.

#2 p 573 a) About $a=0$: $e^{2x} = \sum_{n=0}^{\infty} \frac{(2x)^n}{n!}$ valid for all x.

Like #19 p 573 c) About $a=3$: $\frac{1}{21+x} = \sum_{n=0}^{\infty} \left(\frac{x-3}{18}\right)^n$ valid for $|x-3| < 18$

$\frac{1}{21+x} = \frac{1}{18-(x-3)} = \frac{1}{18} \cdot \frac{1}{1-\frac{x-3}{18}}$

5. On the same grid, sketch the curves $r = \sin\theta$ and $r^2 = 3\cos^2\theta$.
The points (r, θ), with θ in radians, of intersection of these two curves are
$\left(\frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)$, $\left(\frac{\sqrt{3}}{2}, \frac{2\pi}{3}\right)$, $(0,0)$.

p466 #44. The graph of the circle with polar equation $r = \sin\xi$ is shown as a dashed line in the figure at the right, while the graph of the equation

$$r^2 = 3\cos^2\xi$$

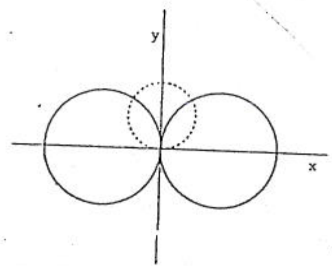
is shown as a solid line. The three points of intersection are

$$r = \frac{1}{2}\sqrt{3}, \quad \xi = \pi/3,$$

$$r = \frac{1}{2}\sqrt{3}, \quad \xi = 2\pi/3,$$

and

$$r = 0.$$



#20 pg 590 4. Working through the steps below, find a power series representation for the given definite integral. For what values of x is this series valid? Express your answers in closed form (i. e. using the $\sum$-sign).

a) $\dfrac{1}{1-t} = \sum_{n=0}^{\infty} t^n$

b) $\dfrac{1}{1+t^5} = \sum_{n=0}^{\infty} (-t^5)^n = \sum_{n=0}^{\infty} (-1)^n t^{5n}$ valid if $|t^5| < 1 \iff |t| < 1$.

c) $\displaystyle\int_0^x \frac{1}{1+t^5}\,dt = \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n+1}}{5n+1}$ valid for $|x| < 1$.

d) Using the above infinite series and the alternating series error estimate test, approximate $\displaystyle\int_0^{.5} \frac{1}{1+t^5}\,dt$ within 3 decimal places of accuracy. You may leave your answer in the form of a sum of n terms but explain why your choice of n works!

$\displaystyle\int_0^{0.5} \frac{1}{1+t^5}\,dt \approx \frac{1}{2} - \frac{1}{6}\left(\frac{1}{2}\right)^6$

⊛ Show your work below:

$$\int_0^x \frac{dt}{1+t^5} = \int_0^x \sum_{n=0}^{\infty} (-1)^n t^{5n}\,dt = \sum_{n=0}^{\infty} (-1)^n \int_0^x t^{5n}\,dt = \sum_{n=0}^{\infty} (-1)^n \frac{t^{5n+1}}{5n+1}\Big|_{t=0}^{t=x} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{5n+1}}{5n+1}$$

$$\int_0^{.5} \frac{dt}{1+t^5} = \sum_{n=0}^{\infty} (-1)^n \frac{1}{5n+1}\left(\frac{1}{2}\right)^{5n+1}$$ ← an alternating Series!

$$= \underset{n=0}{\frac{1}{2}} - \underset{n=1}{\frac{1}{6}\left(\frac{1}{2}\right)^6} + \underset{n=2}{\frac{1}{11}\left(\frac{1}{2}\right)^{11}} - \cdots$$

.0026

.000044
4 places even