MATH 142 FALL 91 EXAM # 4 Outline to Answers

1a. $\sum \frac{x^n}{n!}$ converges for x in $(-\infty,\infty)$. Ratio Test $\rho = \lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\,\frac{n!}{x^n}\right| = \lim_{n\to\infty}\frac{|x|}{n+1} = 0 < 1$.

1b. $\sum \frac{n!\,x^n}{10^n}$ conv. only for $\boxed{x=0}$, i.e. $\boxed{x\in[0,0]}$. Ratio Test $\rho = \lim_{n\to\infty}\left|\frac{(n+1)!\,x^{n+1}}{10^{n+1}}\cdot\frac{10^n}{n!\,x^n}\right| = \lim_{n\to\infty}\frac{|x|}{10}\cdot(n+1) = \begin{cases} 0<1 & \text{if } x=0 \\ \infty>1 & \text{if } x\neq 0.\end{cases}$

1c. $\sum \frac{(2x-6)^n}{5^n} \equiv \sum \frac{2^n(x-3)^n}{5^n}$ conv. for $x \in \boxed{\left(\frac{1}{2},\frac{11}{2}\right)}$

$\frac{1}{2}$ 3 $5\frac{1}{2}$

Root and Ratio Test both give $\rho = \lim_{n\to\infty}\left|\frac{2(x-3)}{5}\right| = \frac{2|x-3|}{5} < 1 \iff |x-3| < \frac{5}{2} = 2\frac{1}{2}$

Check end pts: $x=\frac{11}{2}$, have $\sum 1$ which divg. when $x=\frac{1}{2}$, have $\sum(-1)^n$ which divg.

2a. $e^t = \sum_{n=0}^{\infty}\frac{t^n}{n!} = 1+t+\frac{t^2}{2!}+\cdots$, 2b. $1-e^t = 1-\sum_{n=0}^{\infty}\frac{t^n}{n!} = \sum_{n=1}^{\infty}\frac{-t^n}{n!} = -t-\frac{t^2}{2!}-\cdots$

2c. $\frac{1-e^t}{t} = \sum_{n=1}^{\infty}\frac{-t^n}{t(n!)} = \sum_{n=1}^{\infty}\frac{-t^{n-1}}{n!} = -1-\frac{t^1}{2!}-\frac{t^2}{3!}-\cdots$ valid for all t so valid for all x

2d. $\int_0^x \frac{1-e^t}{t}\,dt = \int_0^x \sum_{n=1}^{\infty}\frac{-t^{n-1}}{n!}\,dt = \sum_{n=1}^{\infty}\int_0^x\frac{-t^{n-1}}{(n-1)!}\,dt = \sum_{n=1}^{\infty}\frac{-1}{(n-1)!}\int_0^x t^{n-1}\,dt = \sum_{n=1}^{\infty}\frac{-1}{(n-1)!}\frac{t^n}{n}\Big|_{t=0}^{t=x}$

$= \sum_{n=1}^{\infty}\frac{-1}{(n-1)!}\left[\frac{x^n}{n}-0\right] = \boxed{\sum_{n=1}^{\infty}\frac{-x^n}{(n-1)!\,n}}$

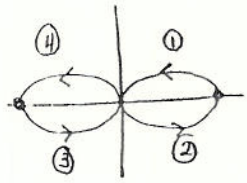
③ Use alternating Series Estimate test: not enough ← 1 place ← $.008 \approx$; $\approx .0002$ 3 places enough.

3. $\sin x = \sum_{n=0}^{\infty}\frac{(-1)^n x^{2n+1}}{(2n+1)!}$ for any x so let $x = 1^{\text{rad.}}$ to see $\sin 1 = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!} = 1-\frac{1}{3!}+\frac{1}{5!}-\frac{1}{7!}+\cdots$

so $\boxed{\sin 1^{\text{rad.}} \approx 1-\frac{1}{3!}+\frac{1}{5!}}$ within $\frac{1}{7!}\approx .0002$ so within 3 places so within 2 places.

4. $r^2 = 4\cos\theta$ $\frac{1}{4}$ period $= \frac{1}{4}\,\frac{2\pi}{1} = \frac{\pi}{2}$

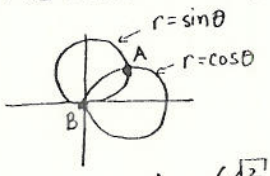
θ	$4\cos\theta$	$r=+\sqrt{\ }$	$r=-\sqrt{\ }$
$0\to\frac{\pi}{2}$	$4\to 0$	① $2\to 0$	③ $-2\to 0$
$\frac{\pi}{2}\to\pi$	$0\to -4$	no pts	no pts
$\pi\to\frac{3\pi}{2}$	$-4\to 0$	no pts	no pts
$\frac{3\pi}{2}\to 2\pi$	$0\to 4$	② $0\to 2$	④ $+0\to -2$



5. $r=\sin\theta$ so $r^2 = r\sin\theta$ so $x^2+y^2 = y$ so circle

$r=\cos\theta$ so $r^2 = r\cos\theta$ so $x^2+y^2 = x$ so circle.

Make chart to see:



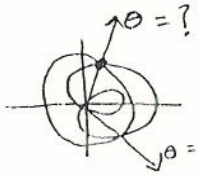
pts of $\cap$: $\boxed{B=(0,0)}$

$\sin\theta = \cos\theta \Rightarrow \theta = \pi/4$ $r = \frac{\sqrt{2}}{2}$

same pt $\theta = \frac{3\pi}{4}$ $r = -\frac{\sqrt{2}}{2}$

So $A = \left(\frac{\sqrt{2}}{2}, \pi/4\right) = \left(-\frac{\sqrt{2}}{2}, \frac{3\pi}{4}\right)$

6. $\theta = ? = \pi/3$



$\theta = -\pi/3$

pt of $\cap$: $2 = 1+2\cos\theta \iff \cos\theta = \frac{1}{2}$ so $\theta = \pi/3$ or $-\pi/3$

$A = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta = \frac{1}{2}\int_{-\pi/3}^{\pi/3}\left[(1+2\cos\theta)^2-(2)^2\right]d\theta \overset{\text{symmetry}}{=} 2\cdot\frac{1}{2}\int_0^{\pi/3}\left[(1+2\cos\theta)^2-(2)^2\right]d\theta$

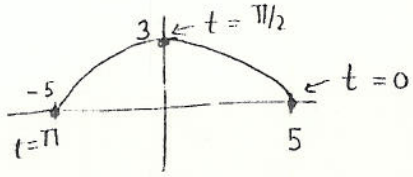
7. $x = 5\cos t \Rightarrow \frac{x}{5} = \cos t$

$y = 3\sin t \Rightarrow \frac{y}{3} = \sin t$

$0\le t\le\pi$

$\cos^2 t + \sin^2 t = 1 \Rightarrow \boxed{\frac{x^2}{25}+\frac{y^2}{9} = 1}$ ellipse upper half

$0\le t\le\pi \Rightarrow t$ is in 1st or 2nd Q $\Rightarrow$

$-1\le\cos t\le 1 \Rightarrow -1\le\frac{x}{5}\le 1 \Rightarrow -5\le x\le 5$

$0\le\sin t\le 1 \Rightarrow 0\le\frac{y}{3}\le 1 \Rightarrow 0\le y\le 3$