

MARK BOX		
PROBLEM	POINTS	
1	3	
2: a - i	27	
TOTAL	30	

NAME: \_\_\_\_\_

class PIN: \_\_\_\_\_

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**INSTRUCTIONS:**

- (1) To receive credit you must:
    - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;**  
such explanations help with partial credit
    - (b) if a line/box is provided, then:
      - show your work BELOW the line/box
      - put your answer on/in the line/box
    - (c) if no such line/box is provided, then box your answer
  - (2) The MARK BOX indicates the problems along with their points.  
Check that your copy of the exam has all of the problems.
  - (3) You may use your notes and the textbook. You cannot use eachother (i.e., you have to take this part solo, without the help of someone else).
  - (4) This exam covers (from *Calculus* by Stewart 6<sup>th</sup> ed,ET): § 11.9, 11.10, 11.11 .
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**Problem Inspiration:** just like the homework.

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**Honor Code Statement**

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : \_\_\_\_\_

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**Due Tuesday Novemeber 22, 2011 at the start of class (12:30pm).**

**No Exceptions!**

1. Using the known Taylor Series

$$\ln(1+t) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} t^n, \quad t \in (-1, 1] \quad , \text{i.e.,} \quad -1 < t \leq 1$$

(as from the handout **Commonly Used Taylor Series**) and methods from Section 11.9, find a power series expansion (in CLOSED form) for

$$y = \ln(x-4) \quad \text{about the center of} \quad x_0 = 5.$$

Hint:  $\ln(x-4) = \ln[1+(x-5)]$ . Also, say when this power series expansion is valid.

$$\ln(x-4) = \sum_{n=1}^{\infty} \boxed{\phantom{(-1)^{n-1}}} (x-5)^n \quad \text{which is valid for} \quad \boxed{\phantom{-1}} < x \leq \boxed{\phantom{1}}.$$

2. Do parts (a) - (i) for the following problem.

$$f(x) = \ln(x - 4) \quad x_0 = 5 \quad J = (4.5, 5.5) .$$

Remark: from problem (1) you know the interval  $J$  can be larger; Prof. G made  $J$  smaller than it can be to make part (i) easier for you.

You might find it easier to do problems (a) - (i) in a different order. Just do what you find easiest.

On parts (a) - (i), use ideas from only Sections 11.10 and 11.11, i.e., use only:

- the definition of Taylor polynomial
- the definition of Taylor series
- the theorem/error-estimate on the  $N^{\text{th}}$ -Remainder term for Taylor polynomials.

Do **NOT** use a known Taylor Series (i.e., do not use methods from Section 11.9, this was problem 1).

- a. Find the following. Note the first column are functions of  $x$  and the second column are numbers.

$f^{(0)}(x) =$	$f^{(0)}(x_0) =$
$f^{(1)}(x) =$	$f^{(1)}(x_0) =$
$f^{(2)}(x) =$	$f^{(2)}(x_0) =$
$f^{(3)}(x) =$	$f^{(3)}(x_0) =$

- b. Find  $N^{\text{th}}$ -order Taylor polynomial of  $y = f(x)$  about  $x_0$  in OPEN form for  $N = 0, 1, 2$ .

$P_0(x) =$
$P_1(x) =$
$P_2(x) =$

- c. Find the Taylor series of  $y = f(x)$  about  $x_0$  in OPEN form.

$$P_{\infty}(x) =$$

- d. Find the Taylor series of  $y = f(x)$  about  $x_0$  in CLOSED form.

$$P_{\infty}(x) =$$

- e. Find the  $n^{\text{th}}$  Taylor coefficient of  $y = f(x)$  about  $x_0$ .

$$c_n =$$

- f. Find the interval  $I$  of convergence of the Taylor series  $y = f(x)$  about  $x_0$ . Recall, the interval of convergence is the set of points for which the series converges, either absolutely or conditionally. (Hint: use the ratio or root test and then check the endpoints.)

$$I =$$

- g. Consider the given interval  $J$  and fix an  $N \in \mathbb{N}$ . Find a good upper bound for the maximum of  $|f^{(N+1)}(c)|$  on the interval  $J$ . Your answer can have an  $N$  in it but it cannot have an:  $x, x_0, c$ . (Note that  $J$  is a subset of  $I$  but Prof. G. might have picked a smaller  $J$  than  $I$  to make the problem easier.)

$$\max_{c \in J} |f^{(N+1)}(c)| \leq$$

- h. Consider the given interval  $J$  and fix an  $N \in \mathbb{N}$ . For each  $x \in J$ , find a good upper bound for the maximum of  $|R_N(x)|$ . Your answer can have an  $N$  and  $x$  in it but it cannot have an:  $x_0, c$ .

$$|R_N(x)| \leq$$

- i. **Carefully** show that  $f(x) = P_\infty(x)$  for each  $x$  in the given interval  $J$  by using part (h) and showing that  $\lim_{N \rightarrow \infty} |R_N(x)| = 0$  for each  $x \in J$ .