

MARK BOX		
PROBLEM	POINTS	
1 a–k	35	
2	10	
3	10	
4	10	
5	10	
6	10	
7	15	
%	100	

NAME: _____

PIN: _____

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat unless you have permission. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) If you do not make at least 17.5 out of the 35 points on Problem 1, then your score for the entire exam will be whatever you made on Problem 1.
- (6) This exam covers (from *Calculus* by Stewart, 6th ed., ET): 11.2–11.7 .

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

I understand that if it is determined that I used any unauthorized assistance or otherwise violated the University's Honor Code then I will receive a failing grade for this course and be referred to the academic Dean and the Office of Academic Integrity for additional disciplinary actions.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : _____

1. Fill-in-the blanks/boxes. All series \sum are understood to be $\sum_{n=1}^{\infty}$.

1a. **n^{th} -term test** for an arbitrary series $\sum a_n$.

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n$ _____ .

1b. **Geometric Series** where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ _____
- diverges if and only if $|r|$ _____

1c. **p -series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p _____
- diverges if and only if p _____

1d. **Integral Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\text{_____})$ for each $n \in \mathbb{N}$
- f is a _____ function
- f is a _____ function
- f is a _____ function .

Then $\sum a_n$ converges if and only if _____ converges.

1e. **Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$. (Fill in the blanks with a_n and/or b_n .)

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and \sum _____ converge, then \sum _____ converge.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and \sum _____ diverge, then \sum _____ diverge.

1f. **Limit Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If _____ $< L <$ _____, then $\sum a_n$ converges if and only if _____ .

1g. **Ratio and Root Tests** for arbitrary-termed series $\sum a_n$ with $-\infty < a_n < \infty$.

Let $\rho = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$ or $\rho = \lim_{n \rightarrow \infty} |a_n|^{\frac{1}{n}}$.

- If ρ _____ then $\sum a_n$ converges absolutely.
- If ρ _____ then $\sum a_n$ diverges.
- If ρ _____ then the test is inconclusive.

1h. **Alternating Series Test** for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

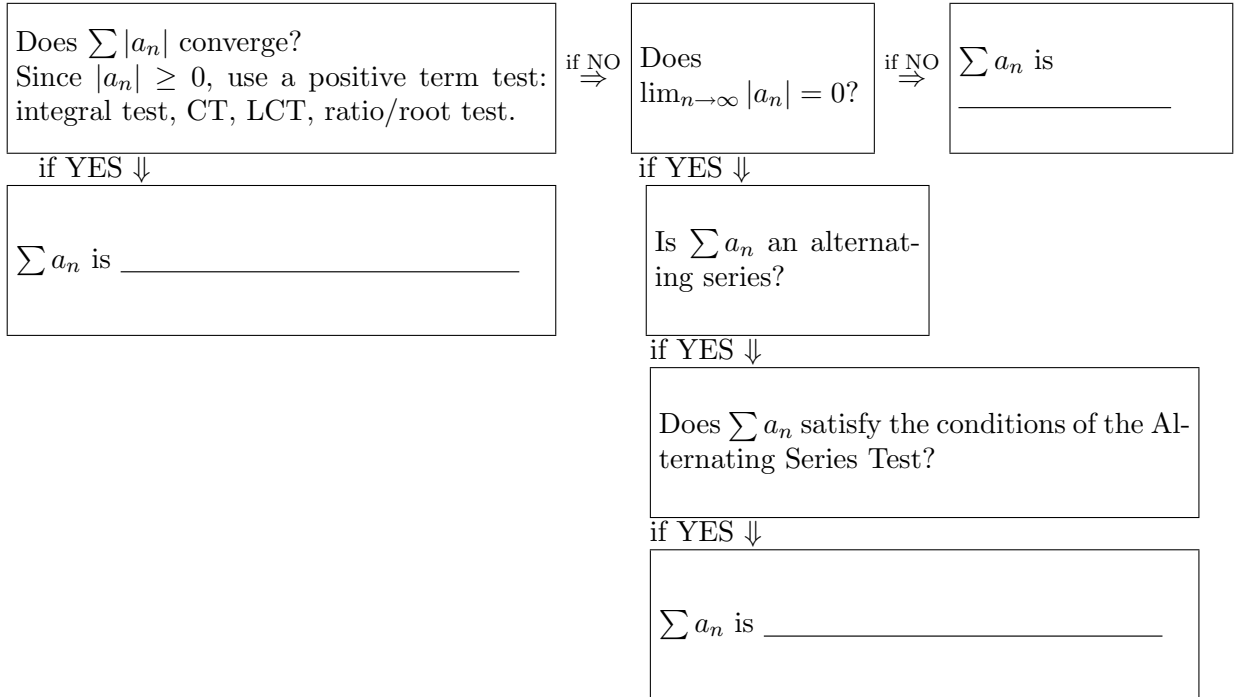
- a_n _____ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n =$ _____

then $\sum (-1)^n a_n$ _____

1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ _____
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ _____ and $\sum |a_n|$ _____
- $\sum a_n$ is divergent if and only if $\sum a_n$ _____

1j. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.



1k. Circle T if the statement is TRUE. Circle F if the statement if FALSE.

- | | | |
|---|---|--|
| T | F | If $\sum a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$. |
| T | F | If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$ converges |
| T | F | If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum (a_n + b_n)$ converges. |
| T | F | If $\sum (a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge. |
| T | F | If $S_N = \sum_{n=1}^N r^n$, then $S_N = \frac{r - r^N}{1 - r}$, for when $r \neq 1$ since we don't like to divide by zero |

2. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$$

absolutely convergent

conditionally convergent

divergent

3. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{4n^7 - n^6 + 21}{11n^8 - n + 17}$$

absolutely convergent

conditionally convergent

divergent

You may use, without showing, that for $n \geq 1$

$$a_n := \frac{4n^7 - n^6 + 21}{11n^8 - n + 17} > 0 .$$

Hint. What would be easier: CT or LCT?

4. Let

$$a_n = \frac{n!}{(2n-1)!}$$

4a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a factorial sign (that is a ! sign) in it.

$\frac{a_{n+1}}{a_n} =$

4b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n-1)!}$$

absolutely convergent

conditionally convergent

divergent

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{3n^2 + 8n - 1}$$

absolutely convergent

conditionally convergent

divergent

6. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{\sqrt{n}}$$

absolutely convergent

conditionally convergent

divergent

7. Geometric Series. (On this page, you should do basic algebra but you do NOT have to do any fancy arithmetic (eg, just making up numbers, you can leave $(\frac{17}{18})^{171}$ as just that.) Let, for $N \geq 51$,

$$s_N = \sum_{n=51}^N 2 \frac{3^{n+1}}{5^n} .$$

- 7a. Do some algebra to write s_N as $\sum_{n=51}^N c r^n$ for an appropriate constant c and ratio r .

$$s_N = \sum_{n=51}^N$$

- 7b. Using the method from class (rather than some formula), find an expression for s_N in closed form (i.e. without a summation \sum sign nor some dots \dots).

$$s_N =$$

- 7c. Does $\sum_{n=51}^{\infty} 2 \frac{3^{n+1}}{5^n}$ converge or diverge? If it converges, find its sum. Justify your answer.

$$\sum_{n=51}^{\infty} 2 \frac{3^{n+1}}{5^n}$$