

If you do not make at least a 50% on this exam's *first few problems*, then your **total** exam score will be only as many points as you managed to get on the *first few problems*. Here are some typical *first few problems*.

1. Fill-in-the blanks/boxes. All series \sum are understood to be $\sum_{n=1}^{\infty}$.

Hint: You should NOT write the words absolute nor conditional on Problem 1!

- 1a. **Sequences** (Afterall, this is needed for Geometric Series!)

Let $-\infty < r < \infty$. (Fill-in-the blanks with *exists* or *does not exist*, i.e. *DNE*)

- If $|r| < 1$, then $\lim_{n \rightarrow \infty} r^n$ _____
- If $|r| > 1$, then $\lim_{n \rightarrow \infty} r^n$ _____
- If $r = 1$, then $\lim_{n \rightarrow \infty} r^n$ _____
- If $r = -1$, then $\lim_{n \rightarrow \infty} r^n$ _____

- 1b. **Geometric Series** where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ _____
- diverges if and only if $|r|$ _____

- 1c. **p -series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p _____
- diverges if and only if p _____

- 1d. **Integral Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\text{_____})$ for each $n \in \mathbb{N}$
- f is a _____ function
- f is a _____ function
- f is a _____ function .

Then $\sum a_n$ converges if and only if _____ converges.

- 1e. **Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.

- 1f. **Limit Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If _____ $< L <$ _____, then $\sum a_n$ converges if and only if _____ .

- 1g. **Ratio and Root Tests** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$.

- If ρ _____ then $\sum a_n$ converges.
- If ρ _____ then $\sum a_n$ diverges.
- If ρ _____ then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum(-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n _____ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n =$ _____

then $\sum(-1)^n a_n$ _____

1i. n^{th} -term test for an arbitrary series $\sum a_n$.

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n$ _____.

1j. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ _____
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ _____ and $\sum |a_n|$ _____
- $\sum a_n$ is divergent if and only if $\sum a_n$ _____

1k. If a power series in $x - x_0$ has radius of convergence R where $0 < R < \infty$, then the power series is:

- absolutely convergent for _____
- divergent for _____

1l. Consider a **function** $y = f(x)$ where $f: [1, \infty) \rightarrow \mathbb{R}$.

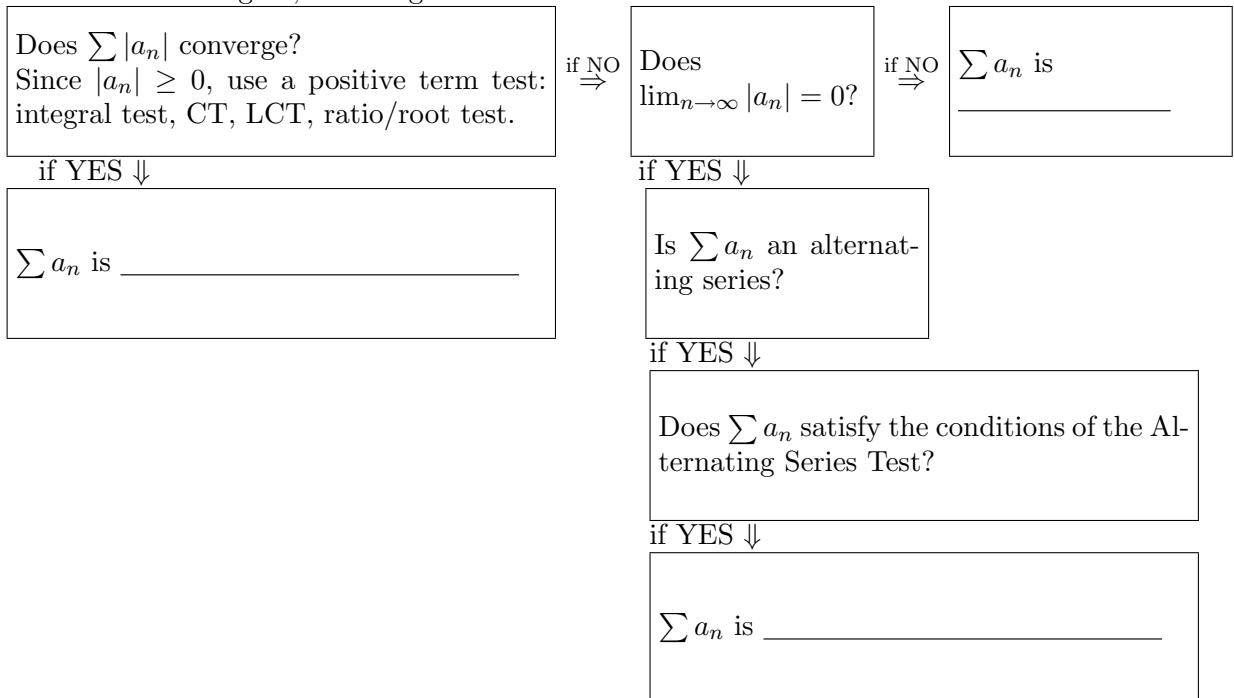
Next consider the corresponding **sequence** $\{a_n\}_{n=1}^{\infty}$ where $a_n \stackrel{\text{def.}}{=} f(n)$.

- If the limit of the **function** $y = f(x)$ as $x \rightarrow \infty$ is L ,

then the limit of the corresponding **sequence** $\{a_n\}_{n=1}^{\infty}$ as $n \rightarrow \infty$ is _____.

- If $\lim_{n \rightarrow \infty} a_n = L$, is it necessarily true that $\lim_{x \rightarrow \infty} f(x) = L$? Circle: **Yes** or **No**

2. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.



3. Circle T if the statement is TRUE. Circle F if the statement is FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.
Scoring: 2 pts for a correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

- | | | |
|---|---|--|
| T | F | If $\sum a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$. |
| T | F | If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$ converges. |
| T | F | If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n \rightarrow \infty} a_n = L$ and $f: [0, \infty) \rightarrow \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{x \rightarrow \infty} f(x) = L$. |
| T | F | If a function $f: [0, \infty) \rightarrow \mathbb{R}$ satisfies that $\lim_{x \rightarrow \infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{n \rightarrow \infty} a_n = L$. |
| T | F | If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges. |
| T | F | If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge. |
| T | F | If $r \neq 1$ and $S_N = \sum_{n=17}^N r^n$, then $S_N = \frac{r^{17} - r^{N+1}}{1 - r}$ for each $N > 17$. |

NOTICE, the above sum starts at $n = 17$, not at $n = 0$.

MORE \Rightarrow

4. Geometric Series. Let, for $N \geq 102$,

$$s_N = \sum_{n=102}^N 3 \frac{2^n}{7^{n+1}} .$$

4a. Do some algebra to write s_N as $\sum_{n=102}^N c r^n$ for an appropriate constant c and ratio r .

$$s_N = \sum_{n=102}^N$$

4b. Using the method from class (rather than some formula), find an expression for s_N in closed form (i.e. without a summation \sum sign nor some dots \dots).

$$s_N =$$

4c. Does $\sum_{n=102}^{\infty} 3 \frac{2^n}{7^{n+1}}$ converge or diverge? If it converges, find its sum. Justify your answer.

$$\sum_{n=102}^{\infty} 3 \frac{2^n}{7^{n+1}}$$