

MARK BOX		
PROBLEM	POINTS	
1 a – j	30	
2	5	
3	10	
4	10	
5 ab	10	
6	10	
7	10	
8	5	
9	10	
%	100	

NAME: _____

PIN: _____

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*; such explanations help with partial credit**
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Stewart, 6th ed., ET): 11.2–11.8 .

Problem Inspiration: Mostly homework and old exam problems. See the solution key for details.

1. Fill-in-the blanks/boxes. All series \sum are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1a. **n^{th} -term test** for an arbitrary series $\sum a_n$.

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n$ _____.

1b. **Geometric Series** where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ _____
- diverges if and only if $|r|$ _____

1c. **p -series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p _____
- diverges if and only if p _____

1d. **Integral Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\text{_____})$ for each $n \in \mathbb{N}$
- f is a _____ function
- f is a _____ function
- f is a _____ function .

Then $\sum a_n$ converges if and only if _____ converges.

1e. **Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.

1f. **Limit Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If _____ $< L <$ _____, then $\sum a_n$ converges if and only if _____.

1g. **Ratio and Root Tests** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$.

- If ρ _____ then $\sum a_n$ converges.
- If ρ _____ then $\sum a_n$ diverges.
- If ρ _____ then the test is inconclusive.

1h. **Alternating Series Test** for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

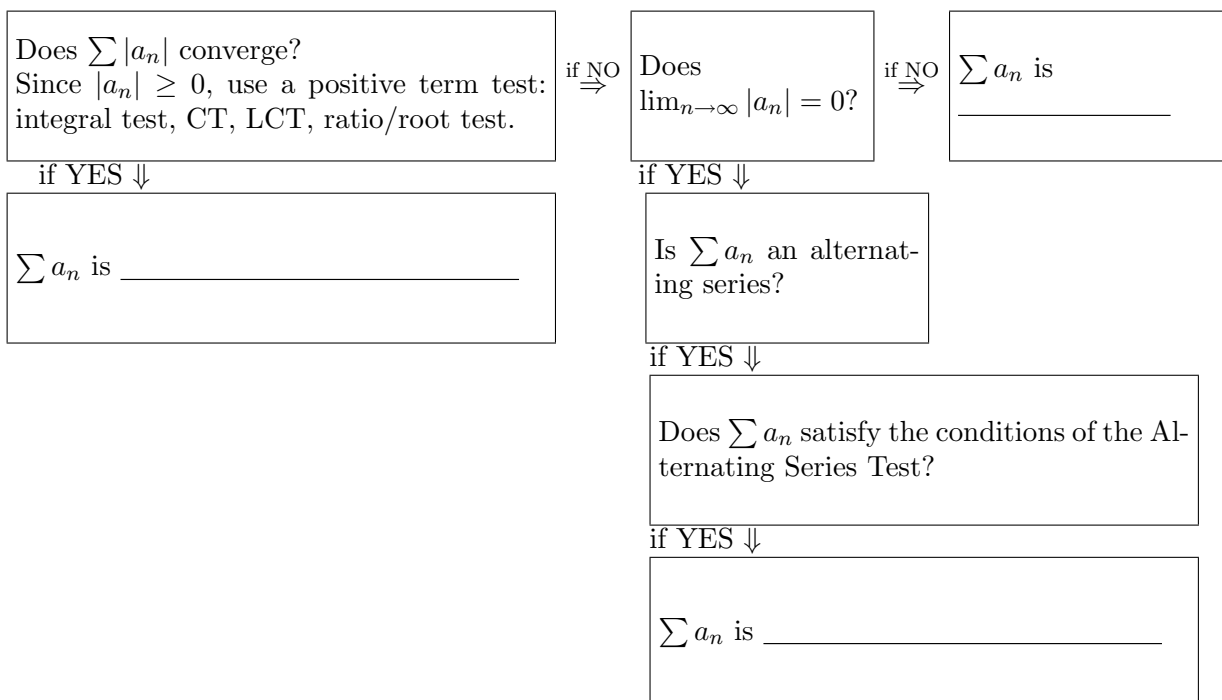
- a_n _____ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n =$ _____

then $\sum (-1)^n a_n$ _____

1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ _____
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ _____ and $\sum |a_n|$ _____
- $\sum a_n$ is divergent if and only if $\sum a_n$ _____

1j. Fill in the 3 blank lines (with absolutely convergent, conditional convergent, or divergent) on the following FLOW CHART for class used to determine if a series $\sum_{n=1}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.



2. Circle T if the statement is TRUE. Circle F if the statement if FALSE.

- | | | |
|---|---|--|
| T | F | If $\lim_{n \rightarrow \infty} a_n = 0$, then $\sum a_n$ converges |
| T | F | If $\sum a_n$ converges, then $\lim_{n \rightarrow \infty} a_n = 0$. |
| T | F | If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges. |
| T | F | If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge. |
| T | F | If $S_N = \sum_{n=1}^N r^n$, then $S_N = \frac{r - r^{N+1}}{1 - r}$. |

3. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=17}^{\infty} \frac{(-1)^n}{n}$$

absolutely convergent

conditionally convergent

divergent

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)(n+2)}}$$

absolutely convergent

conditionally convergent

divergent

5. Let

$$a_n = \frac{n!}{(2n-1)!}$$

5a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a factorial sign (that is a ! sign) in it.

$\frac{a_{n+1}}{a_n} =$

5b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n!}{(2n-1)!}$$

absolutely convergent

conditionally convergent

divergent

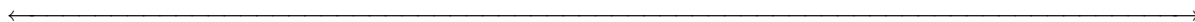
6. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(5x + 10)^n}{n}$$

Hint: $(5x + 10)^n = [5(x + 2)]^n = 5^n(x + 2)^n = 5^n(x - (-2))^n$

The center is $x_0 =$ _____ and the radius of convergence is $R =$ _____ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



7. Consider the formal power series

$$\sum_{n=2}^{\infty} \frac{x^n}{(\ln n)^n}.$$

Hint 1: $\frac{x^n}{(\ln n)^n} = \left[\frac{x}{\ln n}\right]^n$ so would you rather use the root test or the ratio test?

Hint 2: $\ln(a^r) = r \ln(a)$ but $(\ln(a))^r \neq r \ln(a) +$

The center is $x_0 =$ _____ and the radius of convergence is $R =$ _____.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



8. Fill-in the 6 blanks.

Consider the power series

$$\sum_{n=1}^{\infty} (-1)^n a_n x^n$$

where all of the a_n 's are positive. Let's say that you know that

if $0 < x < 17$ then $\sum (-1)^n a_n x^n$ converges

if $x = 17$ then $\sum (-1)^n a_n x^n$ conditionally converges

if $17 < x$ then $\sum (-1)^n a_n x^n$ diverges .

Then this power series has:

center at $x_0 =$ _____ and radius of convergence $R =$ _____ .

Also, what can you say about the following interval? Fill in the blanks below with:

- is absolutely convergent
- is conditionally convergent
- is divergent
- inconclusive (not enough information given to decide in general).

if $-17 < x < 0$ then $\sum (-1)^n a_n x^n$ _____

if $x < -17$ then $\sum (-1)^n a_n x^n$ _____

if $x = 0$ then $\sum (-1)^n a_n x^n$ _____

if $x = -17$ then $\sum (-1)^n a_n x^n$ _____

9. Geometric Series. Let, for $N \geq 102$,

$$s_N = \sum_{n=102}^N 3 \frac{2^n}{7^{n+1}} .$$

9a. Do some algebra to write s_N as $\sum_{n=102}^N c r^n$ for an appropriate constant c and ratio r .

$$s_N = \sum_{n=102}^N$$

9b. Using the method from class (rather than some formula), find an expression for s_N in closed form (i.e. without a summation \sum sign nor some dots \dots).

$$s_N =$$

9c. Does $\sum_{n=102}^{\infty} 3 \frac{2^n}{7^{n+1}}$ converge or diverge? If it converges, find its sum. Justify your answer.

$$\sum_{n=102}^{\infty} 3 \frac{2^n}{7^{n+1}}$$