

MARK BOX		
PROBLEM	POINTS	
a - j	10	
TOTAL	10	

NAME (legibly printed): _____

class PIN: _____

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;**
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show your work **BELOW** the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
 - (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
 - (3) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.): § 10.7, 10.9, 10.10 .
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Problem Inspiration: just like the homework.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : _____

**Due Friday November 13 by 1pm.
Either hand me your paper in class or
slip your paper under my office (LC 309C) door.**

Taylor/Maclaurin Polynomials and Series

Do parts (a) - (j) for the following problem.

$$f(x) = (7 - x)^{-2} \quad x_0 = 5 \quad J = (4, 6) .$$

You might find it easier to do problems (a) - (j) in a different order. Just do what you find easiest.

- On parts (a) - (i), use ideas from only Sections 10.7 and 10.9, i.e., use only:
 - the definition of Taylor polynomial
 - the definition of Taylor series
 - the theorem/error-estimate on the N^{th} -Remainder term for Taylor polynomials.Do **NOT** use a known Taylor Series (i.e., do not use methods from Section 10.10).
- On part (j), the very last part, use a known Taylor Series (as from the handout **Commonly Used Taylor Series**) and methods from Section 10.10.

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- a. Find the following. Note the first column are functions of x and the second column are numbers.

$f^{(0)}(x) =$	$f^{(0)}(x_0) =$
$f^{(1)}(x) =$	$f^{(1)}(x_0) =$
$f^{(2)}(x) =$	$f^{(2)}(x_0) =$
$f^{(3)}(x) =$	$f^{(3)}(x_0) =$
$f^{(4)}(x) =$	$f^{(4)}(x_0) =$

- b. Find N^{th} -order Taylor polynomial of $y = f(x)$ about x_0 in OPEN form for $N = 0, 1, 2, 3, 4$.

$P_0(x) =$
$P_1(x) =$
$P_2(x) =$
$P_3(x) =$
$P_4(x) =$

- c. Find the Taylor series of $y = f(x)$ about x_0 in OPEN form.

$$P_{\infty}(x) =$$

- d. Find the Taylor series of $y = f(x)$ about x_0 in CLOSED form.

$$P_{\infty}(x) =$$

- e. Find the n^{th} Taylor coefficient of $y = f(x)$ about x_0 .

$$c_n =$$

- f. Find the interval I of convergence of the Taylor series $y = f(x)$ about x_0 . Recall, the interval of convergence is the set of points for which the series converges, either absolutely or conditionally. (Hint: use the ratio or root test and then check the endpoints.)

$$I =$$

- g. Consider the given interval J and fix an $N \in \mathbb{N}$. Find a good upper bound for the maximum of $|f^{(N+1)}(c)|$ on the interval J . Your answer can have an N in it but it cannot have an: x, x_0, c . (Note that J is a subset of I but Prof. G. might have picked a smaller J than I to make the problem easier.)

$$\max_{c \in J} |f^{(N+1)}(c)| \leq$$

- h. Consider the given interval J and fix an $N \in \mathbb{N}$. For each $x \in J$, find a good upper bound for the maximum of $|R_N(x)|$. Your answer can have an N and x in it but it cannot have an: x_0, c .

$$|R_N(x)| \leq$$

- i. **Carefully** show that $f(x) = P_\infty(x)$ for each x in the given interval J by using part (h) and showing that $\lim_{N \rightarrow \infty} |R_N(x)| = 0$ for each $x \in J$.

