

MARK BOX		
PROBLEM	POINTS	
1	20	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
take home	10	
%	100	

NAME: _____

CLASS PIN: _____

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*; such explanations help with partial credit**
 - (b) if a line/box is provided, then:
 - show you work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
 - (a) Sections 10.1 - 10.6, 10.8 for the inclass problems
 - (b) whole of Ch 10 for in class fill-in-blank and true/false problems
 - (c) Section 10.7. 10.9, 10.10 for the take home part.

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

Signature : _____

1. Fill-in-the blanks/boxes. All series \sum are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1a. Sequences Let $-\infty < r < \infty$. (Fill-in-the blanks with *exists* or *does not exist*, i.e. *DNE*)

- If $|r| < 1$, then $\lim_{n \rightarrow \infty} r^n$ _____
- If $|r| > 1$, then $\lim_{n \rightarrow \infty} r^n$ _____
- If $r = 1$, then $\lim_{n \rightarrow \infty} r^n$ _____
- If $r = -1$, then $\lim_{n \rightarrow \infty} r^n$ _____

1b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ _____
- diverges if and only if $|r|$ _____

1c. p -series where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p _____
- diverges if and only if p _____

1d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\text{_____})$ for each $n \in \mathbb{N}$
- f is a _____ function
- f is a _____ function
- f is a _____ function .

Then $\sum a_n$ converges if and only if _____ converges.

1e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.

1f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If _____ $< L <$ _____, then $\sum a_n$ converges if and only if _____ .

1g. Ratio and Root Tests for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$.

- If ρ _____ then $\sum a_n$ converges.
- If ρ _____ then $\sum a_n$ diverges.
- If ρ _____ then the test is inconclusive.

1h. Alternating Series Test for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n _____ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n =$ _____

then $\sum (-1)^n a_n$ _____

1i. n^{th} -term test for an arbitrary series $\sum a_n$.

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n$ _____ .

2. Circle T if the statement is TRUE. Circle F if the statement is FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.
Scoring: 2 pts for a correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

- | | | |
|---|---|--|
| T | F | If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n \rightarrow \infty} a_n = L$ and $f: [0, \infty) \rightarrow \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{x \rightarrow \infty} f(x) = L$. |
| T | F | If a function $f: [0, \infty) \rightarrow \mathbb{R}$ satisfies that $\lim_{x \rightarrow \infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{n \rightarrow \infty} a_n = L$. |
| T | F | If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges. |
| T | F | If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge. |
| T | F | If $r \neq 1$ and $S_N = \sum_{n=17}^N r^n$, then $S_N = \frac{r^{17} - r^{N+1}}{1 - r}$ for each $N > 17$. |

NOTICE, the above sum starts at $n = 17$, not at $n = 0$.

3. For the following **SEQUENCES**:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE.

Put your ANSWER IN the box and show your WORK BELOW the box.

3a.

$$\lim_{n \rightarrow \infty} \frac{(4n + 1)(5n + 2)}{17n^2} =$$

3b.

$$\lim_{n \rightarrow \infty} (-1)^n \frac{(4n + 1)(5n + 2)}{17n^2} =$$

3c.

$$\lim_{n \rightarrow \infty} (0.999999917)^n =$$

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

absolutely convergent

conditionally convergent

divergent

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} \frac{2^n 3^n}{n^n}$$

absolutely convergent

conditionally convergent

divergent

Hint: $\frac{2^n 3^n}{n^n} = \left(\frac{2*3}{n}\right)^n$

6. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=17}^{\infty} \frac{1}{(\ln n)^4}$$

absolutely convergent

conditionally convergent

divergent

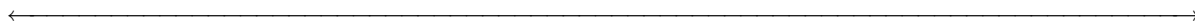
Hint: For any $0 < q < \infty$, if n is big enough then $\ln n < n^q$ and so $\frac{1}{(n^q)^4} < \frac{1}{(\ln n)^4}$.

7. Consider the formal power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{(x+1)^n}{n}$$

The center is $x_0 =$ _____ and the radius of convergence is $R =$ _____.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



8a. Let

$$a_n = \frac{x^{2n+1}}{(2n+1)!}$$

Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a factorial sign (that is a ! sign) in it.

$\frac{a_{n+1}}{a_n} =$

8b. Consider the formal power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

The center is $x_0 =$ _____ and the radius of convergence is $R =$ _____ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

