Prof. Girardi | Math 142 | Fall 2009 | 09.24.09 | Exam 1

MARK BOX		
PROBLEM	POINTS	
1	25	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
Extra Credit	5	
%	100	

NAME: Key

class PIN: ______________________

(*) Extra Credit: 5 point for knowing yor PIN number.

INSTRUCTIONS:

(1) To receive credit you must:
  - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
  - (b) if a line/box is provided, then:
    — show you work BELOW the line/box
    — put your answer on/in the line/box
  - (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use an electronic device, a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.8. .

Problem Inspiration: If I told you here, you would know what method to use. So see the solution key, which will be available from the course homepage after the exam.

Hints:

(1) **You can check your answers to the indefinite integrals by differentiating.**
(2) **For more partial credit, box your $u - du$ substitutions.**

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam. Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________

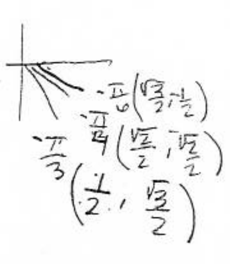
You were warned on the first day of the semester.

1. Fill in the blanks (each worth 1 point).

- $\int \frac{du}{u} = $ ln $|u| + C$
- If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u\,du = $ $\frac{a^u}{\ln a}$ $+ C$
- $\int \cos u\,du = $ $\sin u$ $+C$
- $\int \sin u\,du = $ $-\cos u$ $+C$
- $\int \tan u\,du = $ $\ln|\sec u|$ or $-\ln|\cos u|$ $+C$
- $\int \cot u\,du = $ $\ln|\sin u|$ or $-\ln|\csc u|$ $+C$
- $\int \sec u\,du = $ $\ln|\sec u + \tan u|$ or $-\ln|\sec u - \tan u|$ $+C$
- $\int \csc u\,du = $ $\ln|\csc u - \cot u|$ or $-\ln|\csc u + \cot u|$ $+C$
- $\int \sec^2 u\,du = $ $\tan u$ $+C$
- $\int \sec u \tan u\,du = $ $\sec u$ $+C$
- $\int \csc^2 u\,du = $ $-\cot u$ $+C$
- $\int \csc u \cot u\,du = $ $-\csc u$ $+C$
- If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2}\,du = $ $\frac{1}{a}\tan^{-1}\frac{u}{a}$ $+C$
- If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}}\,du = $ $\sin^{-1}\frac{u}{a}$ $+C$
- If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}}\,du = $ $\frac{1}{a}\sec^{-1}\frac{|u|}{a}$ $+C$
- Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do long division
- Integration by parts formula: $\int u\,dv = $ $uv - \int v\,du$
- Trig substitution: (recall that the *integrand* is the function you are integrating) if the integrand involves a^2+u^2, then one makes the substitution $u = $ $a\tan\theta$
- Trig substitution: if the integrand involves a^2-u^2, then one makes the substitution $u = $ $a\sin\theta$
- Trig substitution: if the integrand involves u^2-a^2, then one makes the substitution $u = $ $a\sec\theta$
- trig formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta) = $ $2\sin\theta\cos\theta$.
- trig formula ... your answer should have $\cos(2\theta)$ in it: $\cos^2(\theta) = \frac{1}{2}($ $1 + \cos 2\theta$ $)$.
- trig formula ... your answer should have $\cos(2\theta)$ in it: $\sin^2(\theta) = \frac{1}{2}($ $1 - \cos 2\theta$ $)$.
- trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is $1 + \tan^2\theta = \sec^2\theta$.

$S + C$

- $\arcsin(-\frac{1}{2}) = $ $-\frac{\pi}{6}$ **RADIANS**. (your answer should be an angle)



a "warm-up" problem

2. $\boxed{\int e^{17x}\,dx = \frac{1}{17}e^{17x} + C}$

$\boxed{\begin{aligned} u &= 17x \\ du &= 17\,dx \end{aligned}} = \frac{1}{17}\int e^{u}\,du$

$= \left(\frac{1}{17}\right) e^{u} + C$

$= \boxed{\frac{1}{17}e^{17x} + C}$

$\boxed{\begin{array}{l} \text{check:} \\ \quad e^{17x}\,\cancel{(17)}\left(\frac{1}{\cancel{17}}\right) = e^{17x} \quad \checkmark \end{array}}$

<u>or</u> $u = e^{17x}$

$du = 17e^{17x}\,dx$

$\int e^{17x}\,dx = \frac{1}{17}\int du = \frac{1}{17}u + c = \frac{1}{17}e^{17x}$

Homework § 8.2 # 1

3. $\boxed{\int xe^x\,dx = xe^x - e^x \quad \text{or} \quad (x-1)e^x \qquad + C}$

$$\boxed{\begin{array}{ll} u = x & dv = e^x dx \\ du = dx & v = e^x \end{array}}$$

$$= xe^x - \int e^x dx$$

$$= \boxed{xe^x - e^x + C}$$

check =

$$\boxed{xe^x + \cancel{e^x} - \cancel{e^x} = xe^x} \checkmark$$

Homework § 8.2 # 33

4. $\int \ln(x+2)\,dx = \boxed{(x+2)\ln(x+2) - x \qquad + C}$

$$\boxed{\begin{array}{ll} \text{let } u = \ln(x+2) & dv = dx \\ du = \frac{1}{x+2}\,dx & v = x \end{array}}$$

$x\ln(x+2) - \int \frac{x}{x+2}\,dx$

$x\ln(x+2) - \int \frac{x \boxed{+2-2}}{x+2}\,dx$

like adding 0
but helps make
easier to integrate

OR

$\frac{x}{x+2} = 1 + \frac{-2}{x+2}$

$$\begin{array}{r} 1 \\ x+2 \,\overline{)\, x} \\ -(x+2) \\ \hline -2 \end{array}$$

$x\ln(x+2) - \int \frac{x+2}{x+2} - \frac{2}{x+2}\,dx$

$x\ln(x+2) - \int \left(1 - \frac{2}{x+2}\right)dx$

$x\ln(x+2) - x + 2\int \frac{1}{x+2}\,dx$

$\underbrace{x\ln(x+2)} - x + \underbrace{2\ln(x+2)} + C$

$(x+2)\ln(x+2) - x + C$

---

or

$$\boxed{\begin{array}{ll} u = \ln(x+2) & dv = dx \\ du = \frac{dx}{x+2} & v = x+2 \end{array}}$$

$$\int \ln(x+2)\,dx = (x+2)\ln(x+2) - \int \frac{x+2}{x+2}\,dx$$

$$= (x+2)\ln(x+2) - \int 1\,dx$$

$$= (x+2)\ln(x+2) - x + C.$$

100 Integrals - # 35

5. $$\boxed{\int \sec^3 x \tan^3 x \, dx = \frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C}$$

$$\boxed{\begin{aligned} &\text{let } s = \sec x \\ &ds = \sec x \tan x \, dx \end{aligned}} \qquad \boxed{\begin{aligned} &\text{let } t = \cancel{\tan x} \\ &\cancel{dt} = \sec^2 x \end{aligned}}$$

doesn't work.

$$\int \sec^2 x \tan^2 x \, [\sec x \tan x \, dx]$$

$$\int \sec^2 x (\sec^2 x - 1) [\sec x \tan x \, dx]$$

$$\int (\sec^4 x - \sec^2 x) [\sec x \tan x \, dx]$$

$$\int (s^4 - s^2) \, ds$$

$$\frac{1}{5} s^5 - \frac{1}{3} s^3 + C$$

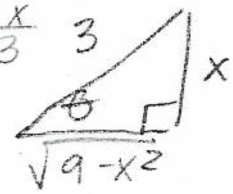
$$\frac{1}{5}\sec^5 x - \frac{1}{3}\sec^3 x + C$$

100 Integrals #26

6. $$\boxed{\int \frac{x^2}{\sqrt{9-x^2}}\,dx = \frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{x\sqrt{9-x^2}}{2} + C}$$

$$\boxed{\begin{aligned} \text{let } x &= 3\sin\theta \\ dx &= 3\cos\theta\, d\theta \end{aligned}}$$

$\sin\theta = \frac{x}{3}$



$\theta = \sin^{-1}\left(\frac{x}{3}\right)$

$\frac{9\cdot\cancel{3}}{\cancel{3}}$

$$\int \frac{9\sin^2\theta\; 3\cos\theta\, d\theta}{\sqrt{9-9\sin^2\theta}}$$

$$9\int \frac{\sin^2\theta\cos\theta\, d\theta}{\sqrt{1-\sin^2\theta}}$$

$$9\int \frac{\sin^2\theta\,\cancel{\cos\theta}\, d\theta}{\cancel{\cos\theta}}$$

$$9\int \sin^2\theta\, d\theta$$

$$9\int \frac{1-\cos 2\theta}{2}\, d\theta$$

$$\frac{9}{2}\int (1-\cos 2\theta)\, d\theta$$

$$\frac{9}{2}\theta - \frac{9}{2}\int \cos 2\theta\, d\theta$$

$$\frac{9}{2}\theta - \frac{9}{2}\cdot\frac{1}{2}\sin 2\theta + C$$

$$\frac{9}{2}\theta - \frac{9}{4}\sin 2\theta + C$$

$$\frac{9}{2}\theta - \frac{9}{2}\sin\theta\cos\theta + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{\cancel{9}}{2}\left[\frac{x}{\cancel{3}}\cdot\frac{\sqrt{9-x^2}}{\cancel{3}}\right] + C$$

$$\frac{9}{2}\sin^{-1}\left(\frac{x}{3}\right) - \frac{1}{2}x\sqrt{9-x^2} + C$$

An example from class.

7. $$\boxed{\int \frac{5x^3-3x^2+2x-1}{x^4+x^2}\,dx = 2\ln|x| + \frac{1}{x} + \frac{3}{2}\ln|x^2+1| - 2\tan^{-1}x + K}$$

- bigger bottoms? Yes / No [degree num] $= 3 < 4 =$ [degree den].
- $x^4+x^2 = x^2(x^2+1)$

$x^2 = (x-0)^2 = (\text{linear term})^2$ ← contribute 2 factors

$b^2-4ac = 0^2-4(1)(1) < 0 \Rightarrow (x^2+1)^1 = (\text{irred. quad.})^1$ ← 1 factor

$$\frac{5x^3-3x^2+2x-1}{x^2(x^2+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+1} = \frac{Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2}{x^2(x^2+1)}$$

should always be the same

- $\Rightarrow 5x^3-3x^2+2x-1 = Ax(x^2+1)+B(x^2+1)+(Cx+D)x^2$ $\quad \boxed{x=0} \Rightarrow \boxed{-1=B}$

Equate coefficients (short way) ← do on board for them

$$\begin{array}{lcl} x^3 & : & 5 = A + C \\ x^2 & : & -3 = B + D \\ x & : & \boxed{2 = A} \\ \text{constant} & : & -1 = B \end{array} \qquad \Rightarrow \boxed{C=3} \qquad \Rightarrow \boxed{D=-2}$$

so

$$\int \frac{5x^3-3x^2+2x-1}{x^4+x^2} = \int \left[\frac{2}{x} + \frac{-1}{x^2} + \frac{3x-2}{x^2+1}\right] dx$$

$u = x^2+1$
$du = 2x\,dx$

$$= 2\int \frac{dx}{x} - 1\int x^{-2}\,dx + 3\cdot\frac{1}{2}\int \underbrace{(x^2+1)^{-1}}_{u^{-1}}\underbrace{(2x\,dx)}_{du} - 2\int\frac{dx}{x^2+1}$$

$$= \boxed{2\ln|x| - \frac{x^{-1}}{-1} + \frac{3}{2}\ln|x^2+1| - 2\tan^{-1}x + K}$$

Homework § 8.8 # 57

8. LaPlace Transform (from a homework problem)

A **transform** is a formula that converts, or *transforms*, one function into another function.

Consider a function of t, denoted by $y = f(t)$. The **LaPlace Transform** of this function $y = f(t)$ is a (new) function, namely the function

$$y = \mathcal{L}\{f(t)\}\ (s)\ ,$$

which is a function of s. The formula for the LaPlace Transform of $y = f(t)$ is

$$\mathcal{L}\{f(t)\}\ (s)\ =\ \int_{t=0}^{t=\infty} e^{-st} f(t)\ dt\ . \tag{8}$$

where, in the integral in (8) above, s is treated as a constant.

The LaPlace Transform of the function

$$f(t) = e^{2t}$$

is the function

$$\boxed{\mathcal{L}\{f(t)\}\ (s)\ =\ \frac{1}{s-2} \qquad \text{for } s > 2}$$

Hint: thus, if $f(t) = e^{2t}$, then by equation (8),

$$\mathcal{L}\{f(t)\}\ (s)\ =\ \int_{t=0}^{t=\infty} e^{-st}\ f(t)\ dt\ =\ \int_{t=0}^{t=\infty} e^{-st}\ e^{2t}\ dt$$

$$\int_{t=0}^{t=\infty} e^{-st} e^{2t}\, dt = \int_{t=0}^{t=\infty} e^{(2-s)t}\, dt = \lim_{b\to\infty} \int_{t=0}^{t=b} e^{(2-s)t}\, dt$$

$$\boxed{\begin{array}{l} u = (2-s)t \\ du = (2-s)dt \end{array}}$$

$$\lim_{b\to\infty} \left. \frac{e^{(2-s)t}}{(2-s)} \right|_{t=0}^{t=b}$$

$$= \lim_{b\to\infty} \left[\frac{e^{(2-s)b}}{(2-s)} - \frac{e^{0}}{(2-s)} \right] = \left[\lim_{b\to\infty} \frac{e^{(2-s)b}}{2-s} + \lim_{b\to\infty} \frac{-1}{(2-s)} \right]$$

$$= \left[\frac{1}{2-s} \lim_{b\to\infty} e^{(2-s)b} \right] + \frac{-1}{2-s}$$

$$= \frac{1}{2-s} \cdot 0 + \frac{-1}{2-s}$$

$$= \frac{1}{s-2}$$

given $s > 2$

so $0 > 2 - s$

so//

if $b \to \infty$

then $(2-s)b \to -\infty$

and $e^{(2-s)b} \to 0$