1. Find and simplify if necessary.

1a. $D_x[e^{3x^2+1}]$

1b. $D_x[\ln(3x^2+17)]$

1c. $D_x[(1+x)^{2x}]$

1d. $D_x[\sin^3(4x)]$

1e. $\frac{d}{dx}e^{\tan x}$

1f. $\frac{d}{dx}[\ln x]^{2x+3}$

1g. $D_x[17^{3x^2+1}]$

1h. $D_x[\ln(\cos(4x))]$

2. Integrate each of the following using an appropriate method.

2a. $\int \ln x\,dx$

2b. $\int \sin^2 x\,dx$

2c. $\int \sin^3 x\,dx$

2d. $\int x^2 \sin x\,dx$

2e. $\int \frac{x^4-2x^2+4x+1}{x^3-x^2-x+1}\,dx$

2f. $\int \frac{x^3}{\sqrt{1-x^2}}\,dx$

2g. $\int x^2 \arctan x\,dx$

2h. $\int e^x \cos x\,dx$

2i. $\int \frac{x}{x^4+4x^2+8}\,dx$

2j. $\int \frac{x^4+2x+2}{x^5+x^4}\,dx$

2k. $\int \frac{x^2}{\sqrt{4-x^2}}\,dx$

2l. $\int_0^\infty \frac{dx}{1+x}$

2m. $\int_{-\infty}^\infty \frac{x}{x^2+1}\,dx$

3. Find the limit.

3a. $\lim_{x\to\infty} x^{\frac{1}{x}}$

3b. $\lim_{n\to\infty} \frac{12n^{17}+188n^7-19n}{4n^{18}-n^9+10}$

3c. $\lim_{x\to\infty}[1+\frac{c}{x}]^x$ where c is a constant and $c \neq 0$

3d. $\lim_{n\to\infty} \frac{n^{17,000}}{e^n}$

4. Let

$$s_N = \sum_{n=5}^{N} \frac{8(3^n)}{(4^{n+2})}$$

for $N = 5, 6, 7, \ldots$. Find a formula for s_N as we did in class (Thus backing up your formula with algebra. Your formula should not have a $\sum$ sign in it nor have $\ldots$ in it.) Does the infinite series $\sum_{n=5}^{\infty} \frac{8(3^n)}{(4^{n+2})}$ converge or diverge? If it converges, find its sum.

5. Decide if the given series is: absolutely convergent, conditionally convergent, or divergent.

5a. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n}}$

5b. $\sum_{n=1}^{\infty}(-1)^n \frac{(3^n)\ n!}{(2n)!}$

5c. $\sum_{n=1}^{\infty}(-1)^n \frac{n}{1+n^2}$

5d. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{5n}$

5e. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n}{10n+1}$

5f. $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n \ln n}$

5g. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n^4}{2^n}$

5h. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{n}{n^2+1}$

5i. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

5j. $\sum_{n=1}^{\infty}(-1)^n \frac{\sin n}{n\sqrt{n}}$

5k. $\sum_{n=1}^{\infty}(-1)^{n+1} \frac{1}{\sqrt{n(n+1)}}$

5l. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{n^2}$

6. Consider the following formal power series. Make a diagram (as we did in class) indicating for which x's this series is: absolutely convergent, conditionally convergent, divergent. Indicate your reasoning. Don't forget to check the endpoints.

6a. $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{x^n}{n}$

6b. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

6c. $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$

6d. $1 + \frac{x-3}{1^2} + \frac{(x-3)^2}{2^2} + \frac{(x-3)^3}{3^2} + \ldots + \frac{(x-3)^{n-1}}{(n-1)^2} + \ldots$

6e. $\sum_{n=1}^{\infty} \frac{(x+1)^n}{\sqrt{n}}$

6f. $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{x^{2n-1}}{2n-1}$

6g. $\sum_{n=1}^{\infty} n!\,(x-1)^n$

7. Recall the geometric series.

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

which is valid for $|r| < 1$.

7a. Using the geometric series, find a power series representation for $f(t) = \frac{1}{9+t}$ about $t = 0$ and say when it is valid.

7b. Using the geometric series, find a power series representation for $g(x) = \frac{7}{9+4x}$ about $x = 3$ and say when it is valid.

8. Express as integral(s) the volume of the solid obtained by revolving the given region R about the given axis of revolution.

8a. R is the region in the first quadrant bounded by the parabola $y^2 = 8x$ and the line $x = 2$. Axis of revolution is the x-axis. (disk/washer method)

8b. R is the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$. Axis of revolution is the $x = 2$. (disk/washer method)

8c. R is the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$. Axis of revolution is the y-axis. (disk/washer method)

8d. R is the region bounded by the parabola $y = 4x - x^2$ and the x-axis. Axis of revolution is the line $y = 6$. (disk/washer method)

8e. R is the region bounded by the parabola $y^2 = 8x$ and the line $x = 2$. Axis of revolution is the line $x = 2$. (shell method)

8f. R is the region bounded by the circle $x^2 + y^2 = 4$. Axis of revolution is the line $x = 3$. (shell method)

8g. R is the region bounded by $y = -x^2 - 3x + 6$ and $x + y - 3 = 0$. Axis of revolution is: (a) the line $x = 3$, and (b) the line $y = 0$. (you choose the method).

9. WORK: For units of let's use in.-lb. where distance is in inches (in.) and force is in pound (lb). Hooke's Law Under appropriate conditions a spring that is stretched x units beyond its natural length pulls back with a force $F(x) = kx$ where k is a (positive) constant (called the spring constant or spring stiffness).

9a. When a particle is located at a distance x inches from the origin, a force of $F(x) = x^2 + 2x$ pounds acts on it. How much work is done in moving it from $x = 1$ to $x = 3$?

9b. A force of 9 pounds is required to stretch a spring from its natural length of 6 inches to a length of 8 inches.

(a) Find the work done in stretching the spring from its natural length to a length of 10 inches.

(b) Find the work done in stretching the spring from a length of 7 inches to a length of 9 inches.

10. Express the length following curves as integral(s).

10a. The curve $y = x^{3/2}$ from $x = 0$ to $x = 5$.

10b. The curve $x = 3y^{3/2} - 1$ from $y = 0$ to $y = 4$.

10c. The arc $24xy = x^4 + 48$ from $x = 2$ to $x = 4$.

10d. The arc of the catenary $y = \frac{1}{2}a(e^{x/a} + e^{-x/a})$ from $x = 0$ to $x = a$.

10e. The curve $x = t^2$, $y = t^3$ from $t = 0$ to $t = 4$.

10f. The cycloid $x = \theta - \sin\theta$, $y = 1 - \cos\theta$ for $\theta = 0$ to $\theta = 2\pi$.