

MARK BOX		
PROBLEM	POINTS	
1	5	
2	5	
3	5	
TOTAL	15	

NAME (legibly printed): Key

class PIN: _____

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;**
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work **BELOW** the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.): § 11.1, 11.2, 11.3 .

Problem Inspiration: just like the homework.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : _____

1. Consider the point, in polar coordinates,

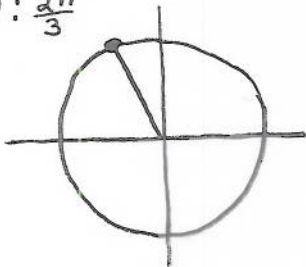
$$P = (r, \theta) = \left(4, \frac{2\pi}{3}\right).$$

In cartesian coordinates, the point P is given by

$$P = (x, y) = (-2, 2\sqrt{3}).$$

Below graph, and CLEARLY label, the following points.

$$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right) : \frac{2\pi}{3}$$



$$P = \left(4, \frac{2\pi}{3}\right)$$

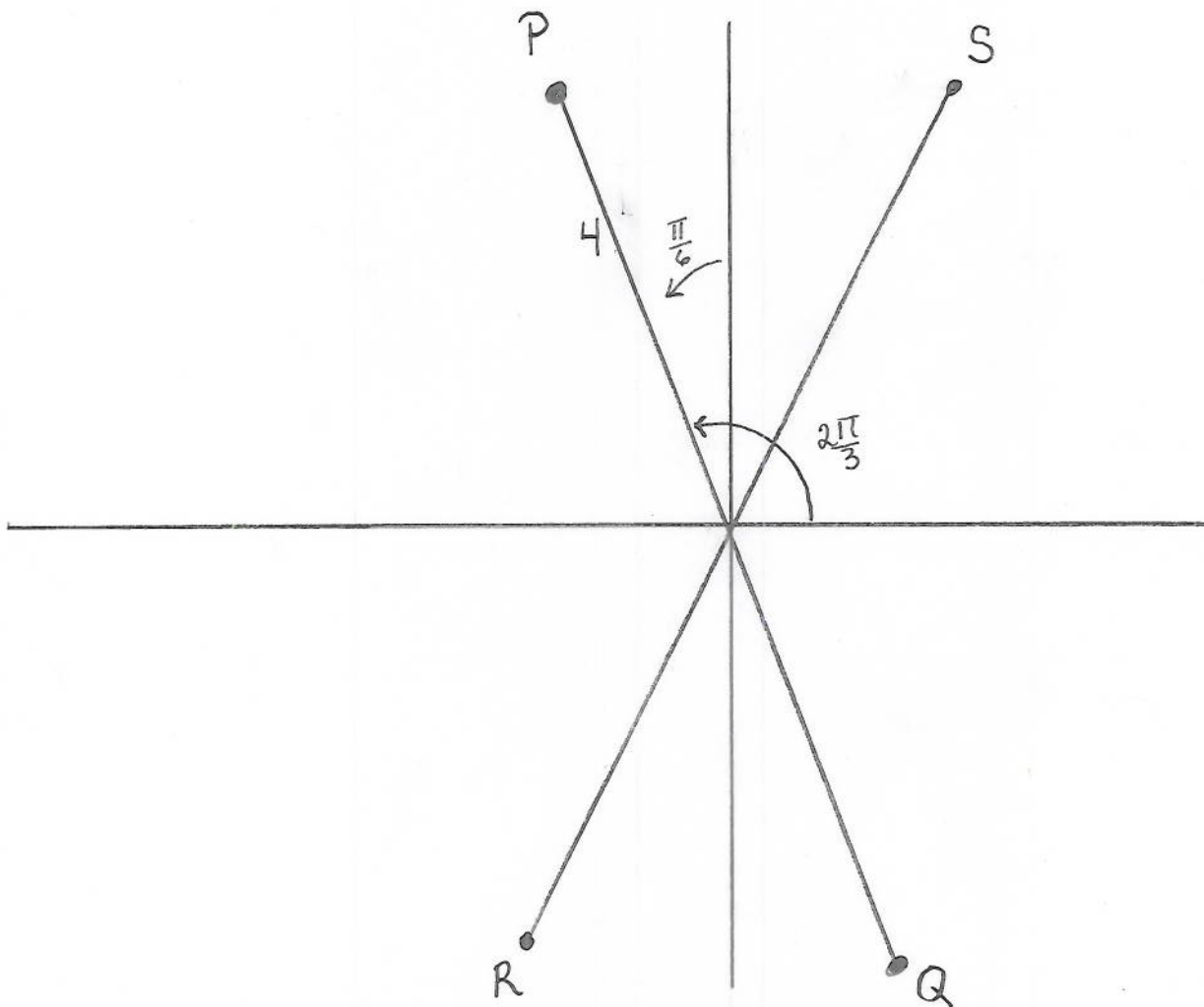
$$Q = \left(-4, \frac{2\pi}{3}\right)$$

$$R = \left(4, -\frac{2\pi}{3}\right)$$

$$S = \left(-4, -\frac{2\pi}{3}\right).$$

$$\begin{aligned} x &= r \cos \theta = 4 \cos \frac{2\pi}{3} \\ &= 4 \left(-\frac{1}{2}\right) = -2 \end{aligned}$$

$$\begin{aligned} y &= r \sin \theta = 4 \sin \frac{2\pi}{3} \\ &= 4 \left(\frac{\sqrt{3}}{2}\right) = 2\sqrt{3} \end{aligned}$$



2. Consider the curve in polar coordinate

$$r^2 = 9 \sin(2\theta)$$

§ 11.1 # 42

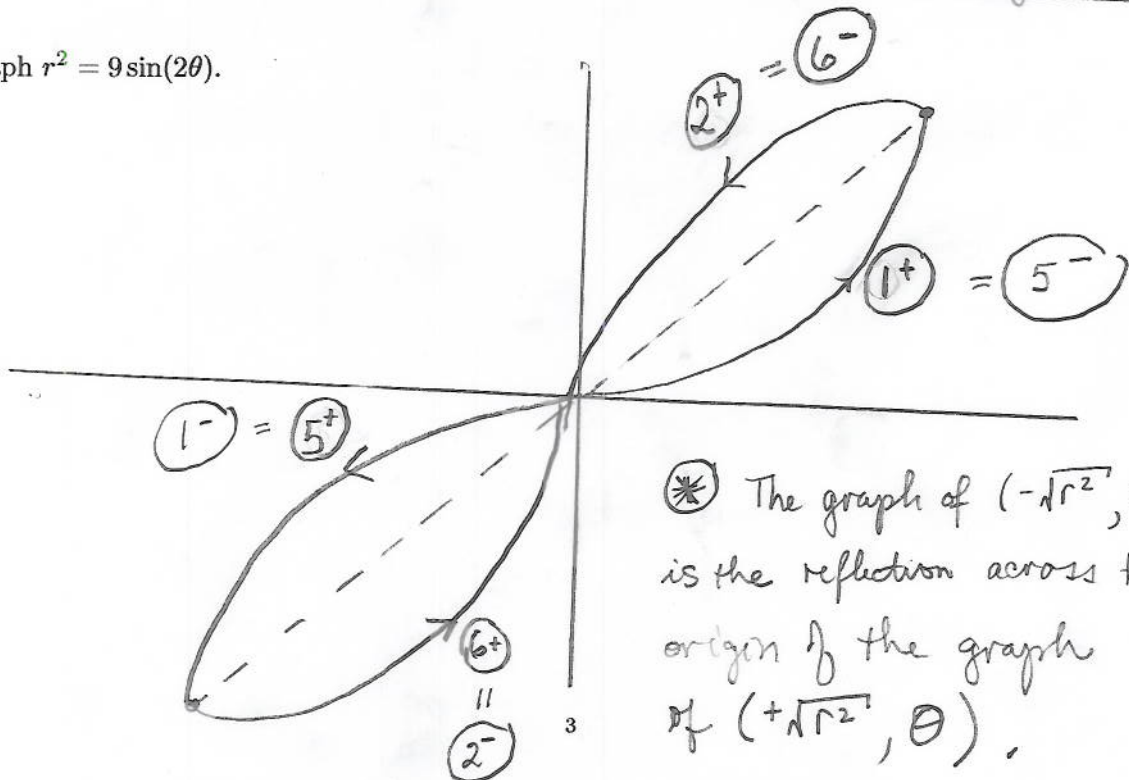
2a. The period of $r^2 = 9 \sin(2\theta)$ is $\frac{2\pi}{2} = \pi$.

2a. $\frac{\text{the period of } r^2 = 9 \sin(2\theta)}{4} = \frac{\pi}{4}$

2c. Make a chart, as we did in class, to help you graph $r^2 = 9 \sin(2\theta)$.

	θ	2θ	$r^2 = 9 \sin 2\theta$	$r = +\sqrt{9 \sin 2\theta}$	$r = -\sqrt{9 \sin 2\theta}$
①	$0 \rightarrow \frac{\pi}{4}$	$0 \rightarrow \frac{\pi}{2}$	$0 \rightarrow 9$	$0 \rightarrow 3$	$0 \rightarrow -3$
②	$\frac{\pi}{4} \rightarrow \frac{\pi}{2}$	$\frac{\pi}{2} \rightarrow \pi$	$9 \rightarrow 0$	$3 \rightarrow 0$	$-3 \rightarrow 0$
③	$\frac{\pi}{2} \rightarrow \frac{3\pi}{4}$	$\pi \rightarrow \frac{3\pi}{2}$	$0 \rightarrow -9$	no graph	no graph
④	$\frac{3\pi}{4} \rightarrow \pi$	$\frac{3\pi}{2} \rightarrow 2\pi$	$-9 \rightarrow 0$	no graph	no graph
⑤	$\pi \rightarrow \frac{5\pi}{4}$	$2\pi \dots$	$0 \rightarrow 9$	$0 \rightarrow 3$	$0 \rightarrow -3$
⑥	$\frac{5\pi}{4} \rightarrow \frac{3\pi}{2}$: just go around unit circle again	$9 \rightarrow 0$	$3 \rightarrow 0$	$-3 \rightarrow 0$
⑦	$\frac{3\pi}{2} \rightarrow \frac{7\pi}{4}$		$0 \rightarrow -9$	no graph	no graph
⑧	$\frac{7\pi}{4} \rightarrow 2\pi$		$-9 \rightarrow 0$	no graph	no graph

2d. Graph $r^2 = 9 \sin(2\theta)$.



* The graph of $(-\sqrt{r^2}, \theta)$ is the reflection across the origin of the graph of $(+\sqrt{r^2}, \theta)$.

3. Express the area enclosed by $r^2 = 9 \sin(2\theta)$ as an integral with respect to θ (ok ... with respect to θ means a $d\theta$ in there). (You do not have to evaluate this integral.)

area =

$$\text{Area} = \frac{1}{2} \int_{\theta=\alpha}^{\theta=\beta} [f(\theta)]^2 d\theta$$

$$= \frac{1}{2} \int_{\theta=\alpha}^{\theta=\beta} 9 \sin(2\theta) d\theta$$

... many answers from here, eg:

$$\text{Area} = 2 \cdot \frac{1}{2} \int_{\theta=0}^{\theta=\frac{\pi}{2}} 9 \sin 2\theta d\theta$$

$$\text{or} = 4 \cdot \frac{1}{2} \int_{\theta=0}^{\theta=\frac{\pi}{4}} 9 \sin 2\theta d\theta$$