Prof. Girardi	Math 142	Fall 2008	11.25.08	Exam 3

MARK BOX		
PROBLEM	POINTS	
sign honor code	2	
correct pin	2	
$1-6$	26	
7abcd	40	
8	10	
9	10	
10	10	
%	100	

NAME (legibly printed): James Bond

class PIN: 007

INSTRUCTIONS:

(1) To receive credit you must:
   (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
   (b) if a line/box is provided, then:
   — show you work BELOW the line/box
   — put your answer on/in the line/box
   (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Sections 7.1, 7.2, 7.3, 7.4, 7.6, 7.7 .

Problem Inspiration: See the answer key.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.
As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.
Furthermore, I have not only read but will also follow the above Instructions.

Signature : ______________________________

Throughout this exam, you need to only set up the integral expressing the asked for quantity. You do not have to integrate your integral. You do not have to do lots of algebra to your integrand.

problem 1–5 : in class quiz
problem 6 : HMWK § 7.7 #5

Problem 1 - 6: Fill-in-the blanks/boxes.

- **In 1a and 2a, fill in the blank with:** **perpendicular or parallel.**
- **In 1b, 1c, 1d, 2b, 2c, fill in the blank with a formula involving** ***some of*:**
  2 , π , radius , radius$_{\text{big}}$, radius$_{\text{little}}$, average radius , height , and/or thickness.

1. Disk/Washer Method

Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the disk or washer method.

1a. You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is parallel to the axis of revolution.

1b. If you use the **disk method**, then the volume of a typical disk is:

$\pi(\text{radius})^2(\text{height})$.

1c. If you use the **washer method**, then the volume of a typical washer is:

$\pi(\text{rad}_{\text{big}})^2(\text{height}) - \pi(\text{rad}_{\text{little}})^2(\text{height})$ or $\pi\left[(\text{rad}_{\text{big}})^2 - (\text{rad}_{\text{little}})^2\right](\text{height})$

1d. If you partition the z-axis, the $\Delta z =$ height .

2. Shell Method

Let's say you revolve some region in the xy-plane around an axis of revolution so you get a solid of revolution. Next you want to find the volume of this solid of revolution using the shell method.

2a. You should partition the coordinate axis (i.e., the x-axis or the y-axis) that is perpendicular to the axis of revolution.

2b. If you use the **shell method**, then the volume of a typical shell is:

$2\pi(\text{average radius})(\text{height})(\text{thickness})$.

2c. If you partition the z-axis, the $\Delta z =$ thickness or $\text{radius}_{\text{big}} - \text{radius}_{\text{little}}$.

3. Arc Length

3a. The arc length L of a smooth curve $y = f(x)$ over the interval $[a, b]$ is defined by the following definite integral.

$$L = \int_{x=a}^{x=b} \sqrt{1+[f'(x)]^2}\,dx \overset{\text{or}}{=} \int_{x=a}^{x=b} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx.$$

3b. The arc length L of a smooth curve $x = g(y)$ over the interval $[c, d]$ is defined by the following definite integral.

$$L = \int_{y=c}^{y=d} \sqrt{1+[g'(y)]^2}\,dy \overset{\text{or}}{=} \int_{y=c}^{y=d} \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy.$$

3c. The arc length L of a curve that is parametrized by

$$x = x(t) \quad , \quad y = y(t) \qquad (a \le t \le b)$$

such that no segment of the curve is traced more than once as t increases from a to b and also $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are continuous functions for $a \le t \le b$, is defined by the following definite integral.

$$L = \int_{x=a}^{x=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

4. Average Value of a Function

If $y = f(x)$ is continuous on the interval $[a, b]$, the the average value f_{ave} of $y = f(x)$ on $[a, b]$ is defined to be

$$f_{\text{ave}} = \boxed{\frac{\int_{x=a}^{x=b} f(x)\,dx}{b-a}}.$$

5. Work

Suppose that Integration-Moose moves in the positive direction along a coordinate line over the interval $[a, b]$ while subjected to a variable force $F(x)$ that is applied in the directins of the motion. The work W preformed by the force on Integration-Moose is defined by the following definite integral.

$$W = \int_{x=a}^{x=b} F(x)\,dx.$$

6. Circle the scenario in which you perform more work:

(1) by rasing a cup of coffee from a table to your mouth

(2) by holding a calculus textbook at shoulder level for 5 minutes.

HMWK Ch 7 Review #6

7. THIS PROBLEM HAS PARTS: 7a, 7b, 7c, 7d. The region R is the same for all 4 parts.

Let R be the region in the first quadrant enclosed by $y = 2x$ and $y = x + 4$ and $x = 0$.

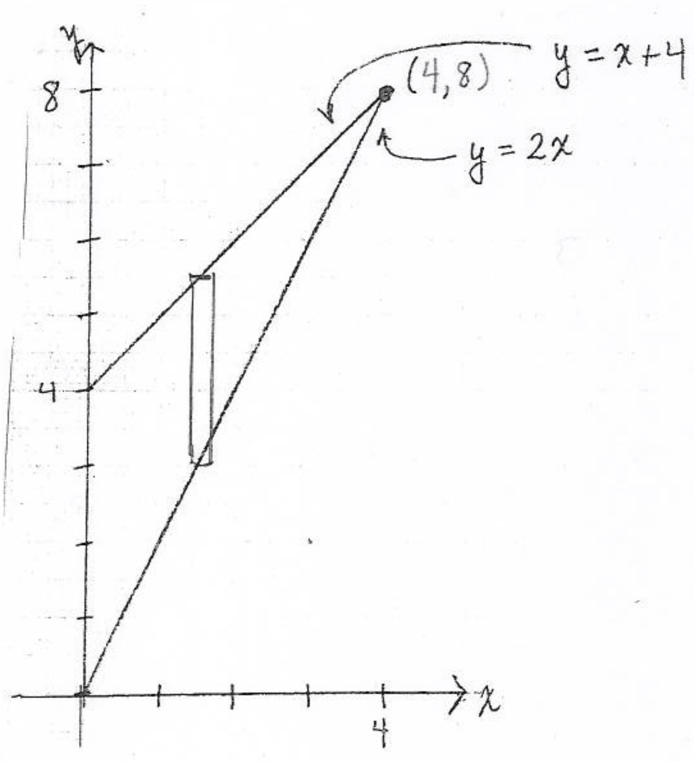
7a. Express the area of R as integral(s) with respect to x.

$$\text{Area} = \int_{x=0}^{x=4} \left[(x+4) - (2x) \right] dx$$

7b. Express the area of R as integral(s) with respect to y.

$$\text{Area} = \int_{y=0}^{y=4} \left[\frac{y}{2} \right] dy + \int_{y=4}^{y=8} \left[\left(\frac{y}{2}\right) - (y-4) \right] dy$$

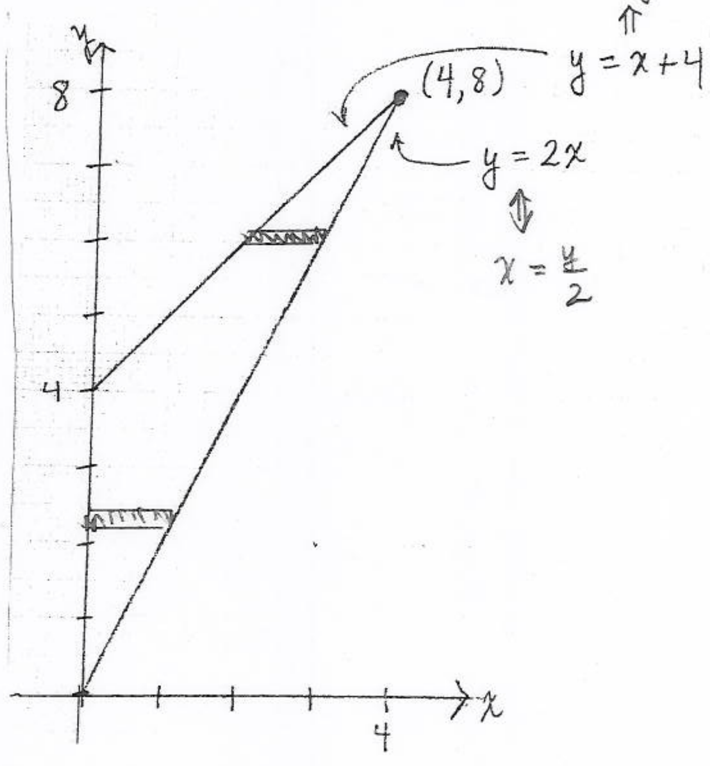
7a w.r.t. x



7b w.r.t. y

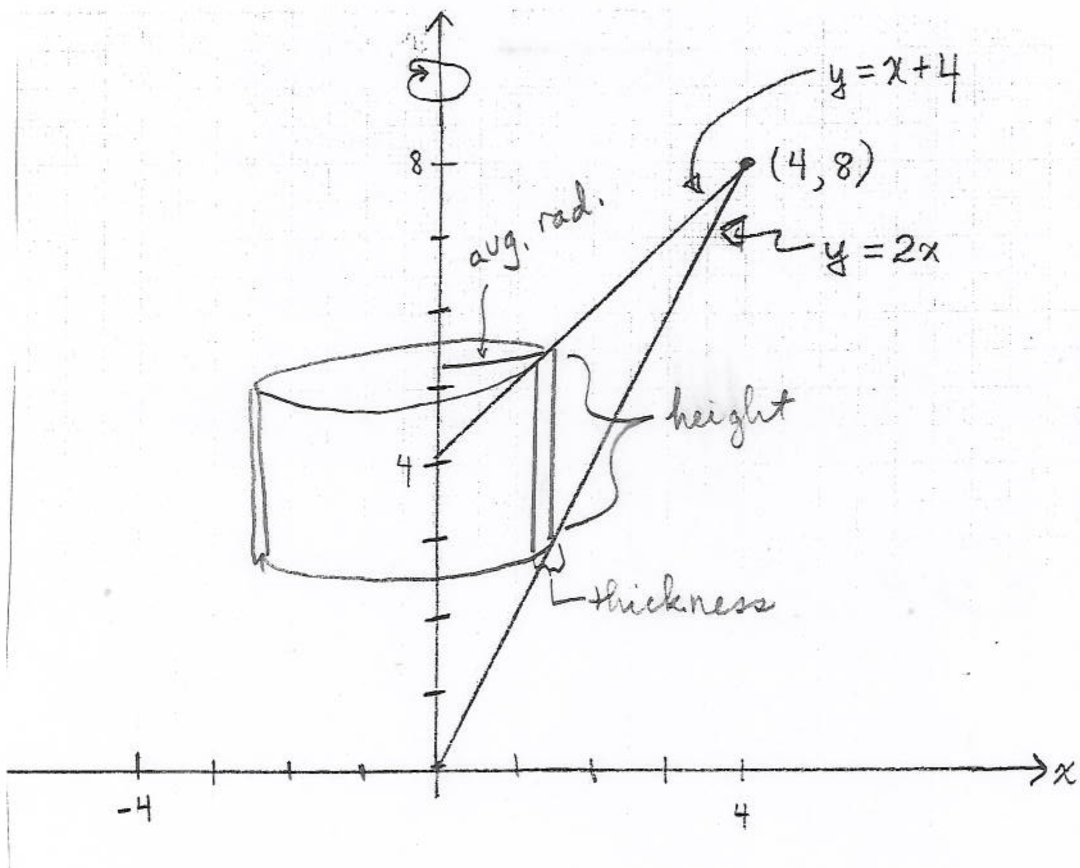
$x = y - 4$

$\Uparrow$



7c. Using the shell method, express as integral(s) the volume of the solid generated by revolving R about the y-axis.

Volume = $\displaystyle\int_{x=0}^{x=4} 2\pi\, x\, [(x+4)-(2x)]\, dx$



Shell $\Rightarrow$ partition axis $\perp$ to axis of revolution $\Rightarrow$ partition $\underline{x\text{-axis}}$

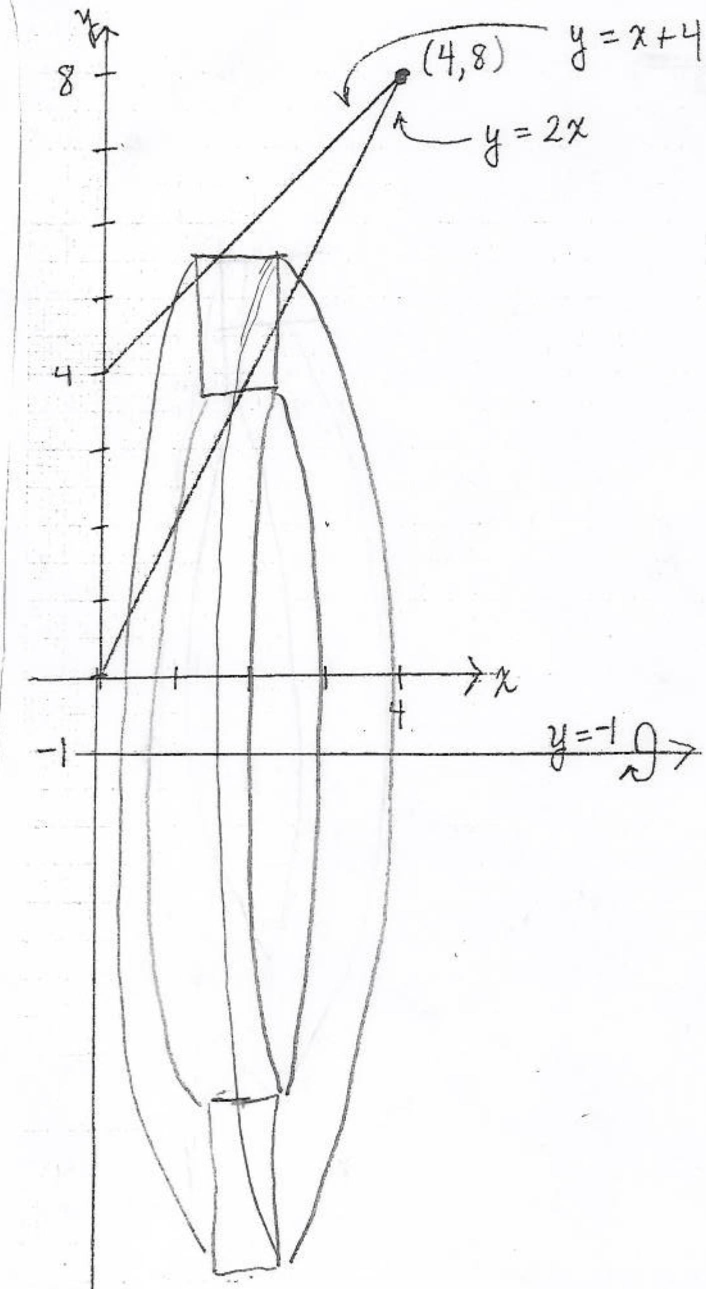
$V_{\text{typical shell}}$ = (2π) (average radius) (height) (thickness)

$= (2\pi) \quad (x) \quad [(x+4)-(2x)] \quad \Delta x$

→ partition axis ∥ to axis of revolution ⇒ partition x-axis
have a "hole" so washer (not disk)

7d. Using the disk/washer method, express as integral(s) the volume of the solid generated by revolving R about the line $y = -1$.

$$\text{Volume} = \int_{x=0}^{x=4} \pi \left[(x+5)^2 - (2x+1)^2 \right] dx$$



Volume typical Washer

= Volume Big Washer
− Volume Little Washer

$= \pi\,(\text{Big Radius})^2\,(\text{height})$
$- \pi\,(\text{little radius})^2\,(\text{height})$

$= \pi\,((x+4)+1)^2\,(\Delta x)$
$- \pi\,((2x)+1)^2\,(\Delta x)$

Fall 06, Exam 1, #4

8 ~~4~~. Express the arclength of the parameterized curve

$$x(t) = t^2 + 4$$
$$y(t) = t + 5$$

from the point

$$P = (4, 5)$$

to the point

$$Q = (13, 8)$$

as an integral with respect to t.

$$\boxed{\text{arclength} = \int_{t=0}^{t=3} \sqrt{(2t)^2 + 1^2}\, dt \ \underset{=}{\text{or}} \int_{t=0}^{t=3} \sqrt{4t^2+1}\, dt}$$

Once again, you do not have to do lots of algebra to your integrand nor integrate your integral.

$$P = (4,5) \iff \begin{bmatrix} 4 = t^2 + 4 \\ 5 = t + 5 \end{bmatrix} \iff t = 0$$

$$Q = (13,8) \iff \begin{bmatrix} 13 = t^2 + 4 \\ 8 = t + 5 \end{bmatrix} \iff t = 3$$

$$\frac{dx}{dt} = \frac{d}{dt}(t^2+4) = 2t$$

$$\frac{dy}{dt} = \frac{d}{dt}(t+5) = 1$$

$$\text{arclength} = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

HMWK § 7.7 # 1

9. Express as an integral the work done when a variable force of $F(x) = x^2$ lb in the positive x-direction moves Integration Moose from $x = 7$ to $x = 17$ ft.

work = $\displaystyle\int_{x=7}^{x=17} x^2 \, dx$ ft-lbs.

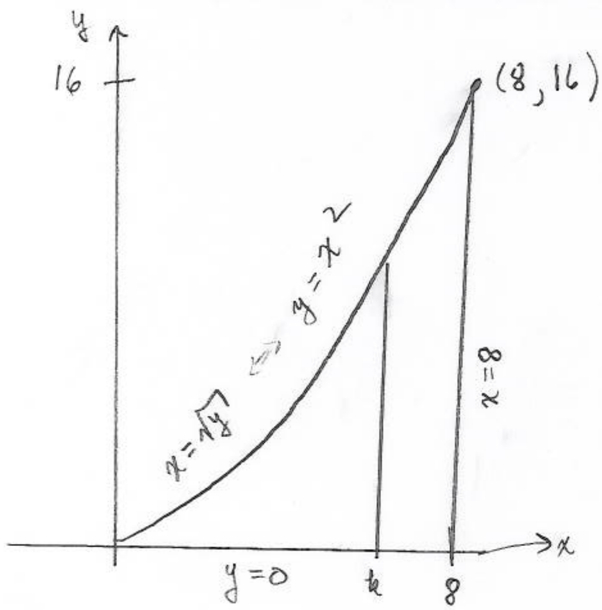
→ HMWK § 7.1 # 31 & 33 . also Fall 2005, Exam 1, # 6.

10. Find a vertical line $x = k$ that divides the area enclosed by

$$x = \sqrt{y} \qquad \text{and} \qquad x = 8 \qquad \text{and} \qquad y = 0$$

into two equal parts. $\Rightarrow x^2 = y$ and $y \geq 0$

ANSWER: the vertical line is $x = \underline{4(2)^{2/3}}$.



Want $\int_0^k x^2\,dx = \int_k^8 x^2\,dx$

$\iff \frac{1}{3}x^3 \Big|_{x=0}^{x=k} = \frac{1}{3}x^3 \Big|_{x=k}^{x=8}$

$\iff \frac{1}{3}\left[k^3 - 0^3\right] = \frac{1}{3}\left[8^3 - k^3\right]$

$\iff k^3 = 8^3 - k^3$

$\iff 2k^3 = 8^3$

$\iff k^3 = \frac{8^3}{2} \iff k = \frac{(8^3)^{1/3}}{2^{1/3}} = \frac{8}{2^{1/3}} = 4 \cdot \frac{2^1}{2^{1/3}} = 4 \cdot 2^{2/3}$