

MARK BOX		
PROBLEM	POINTS	
1 a-j	30	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
take home	10	
%	100	

NAME (legibly printed): _____

class PIN: _____

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning;**
no credit will be given for an answer that *just appears*;
such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show you work **BELOW** the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Sections 10.1 - 10.6, 10.8 for the inclass and Section 10.7. 10.9, 10.10 for the take home. .

Problem Inspiration: See the answer key.

1. Fill-in-the blanks/boxes. All series \sum are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

1a. **n^{th} -term test** for an arbitrary series $\sum a_n$.

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n$ _____.

1b. **Geometric Series** where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ _____
- diverges if and only if $|r|$ _____

1c. **p -series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p _____
- diverges if and only if p _____

1d. **Integral Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\text{_____})$ for each $n \in \mathbb{N}$
- f is a _____ function
- f is a _____ function
- f is a _____ function .

Then $\sum a_n$ converges if and only if _____ converges.

1e. **Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ _____, then $\sum a_n$ _____.

1f. **Limit Comparison Test** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If _____ $< L <$ _____, then $\sum a_n$ converges if and only if _____ .

1g. **Ratio and Root Tests** for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$.

- If ρ _____ then $\sum a_n$ converges.
- If ρ _____ then $\sum a_n$ diverges.
- If ρ _____ then the test is inconclusive.

1h. **Alternating Series Test** for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n _____ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n =$ _____

then $\sum (-1)^n a_n$ _____

1i. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ _____
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ _____ and $\sum |a_n|$ _____
- $\sum a_n$ is divergent if and only if $\sum a_n$ _____

1j. If a power series in $x - x_0$ has radius of convergence R where $0 < R < \infty$, then the power series is:

- absolutely convergent for _____
- divergent for _____

2. Circle T if the statement is TRUE. Circle F if the statement is FALSE. To be more specific: circle T if the statement is always true and circle F if the statement is NOT always true.

Scoring: 2 pts for a correct answer, 1 pt for a blank answer, 0 pts for an incorrect answer.

- | | | |
|---|---|--|
| T | F | If a function $f: [0, \infty) \rightarrow \mathbb{R}$ satisfies that $\lim_{x \rightarrow \infty} f(x) = L$ and $\{a_n\}_{n=1}^{\infty}$ is a sequence satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{n \rightarrow \infty} a_n = L$. |
| T | F | If a sequence $\{a_n\}_{n=1}^{\infty}$ satisfies that $\lim_{n \rightarrow \infty} a_n = L$ and $f: [0, \infty) \rightarrow \mathbb{R}$ is a function satisfying that $f(n) = a_n$ for each natural number n , then $\lim_{x \rightarrow \infty} f(x) = L$. |
| T | F | If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges. |
| T | F | If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge. |
| T | F | If $r \neq 1$ and $S_N = \sum_{n=17}^N r^n$, then $S_N = \frac{r^{17} - r^{N+1}}{1 - r}$ for each $N > 17$. |

NOTICE, the above sum starts at $n = 17$, not at $n = 0$.

3. Find the limit of the following **sequences**. Indicate your reasoning.

Put ANSWER IN BOX and show WORK BELOW BOX.

There is: 3a, 3b, 3c (on this page), and then 3d on the next page.

3a. $\lim_{n \rightarrow \infty} (.999917)^n =$

3b. $\lim_{n \rightarrow \infty} (1.000017)^n =$

3c. $\lim_{n \rightarrow \infty} \frac{5n^3 + 6n + 3}{17n^3 + 9n^2 + 4} =$

3d IS ON NEXT THE PAGE \implies

3d.

$$\lim_{n \rightarrow \infty} \left(\frac{n+6}{n+1} \right)^n =$$

Hint: if $\ln(a_n) \rightarrow 17$, then $a_n \equiv e^{\ln(a_n)} \rightarrow e^{17}$

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^2 + 1}{17n^2 + n - 1}$$

absolutely convergent

conditionally convergent

divergent

5. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{n^3}{n^4 + 1}$$

absolutely convergent

conditionally convergent

divergent

6. Let

$$a_n = \frac{3^n n!}{(2n)!}$$

6a. Find an expression for $\frac{a_{n+1}}{a_n}$ that does NOT have a factorial sign (that is a ! sign) in it.

Hint: $(2(n+1))! = (2n+2)!$

$\frac{a_{n+1}}{a_n} =$

6b. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{3^n n!}{(2n)!}$$

absolutely convergent

conditionally convergent

divergent

7. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+6)^n}{4^n}$$

Hint: $(2x+6)^n = [2(x+3)]^n = 2^n(x+3)^n = 2^n(x-(-3))^n$

The center is $x_0 =$ _____ and the radius of convergence is $R =$ _____ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

