

MARK BOX	
PROBLEM	POINTS
1 a-y	25
2	10
3	10
4	10
5	10
6	10
7	10
take home	10
Extra Credit	5
%	100

NAME (legibly printed): James Bond

class PIN: 007

(* Extra Credit: 5 point for knowing your PIN number.

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that just appears;** such explanations help with partial credit
 - (b) if a line/box is provided, then:
 - show your work BELOW the line/box
 - put your answer on/in the line/box
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.): Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.7, 8.8. .

Problem Inspiration: If I told you here, you would know what method to use. So see the solution key, which will be available from the course homepage shortly after the exam.

Hints:

- (1) You can check your answers to the indefinite integrals by differentiating.
- (2) + C
- (3) For more partial credit, box your $u - du$ substitutions.

Similar to problem #1 from last several semester
I taught this class.

1. Fill in the blanks (each worth 1 point).

1a. $\int \frac{du}{u} = \ln |u| + C$

1b. If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u du = \frac{a^u}{\ln a} + C$

1c. $\int \cos u du = \sin u + C$

1d. $\int \sec^2 u du = \tan u + C$

1e. $\int \sec u \tan u du = \sec u + C$

1f. $\int \sin u du = -\cos u + C$

1g. $\int \csc^2 u du = -\cot u + C$

1h. $\int \csc u \cot u du = -\csc u + C$

1i. $\int \tan u du = -\ln |\cos u| + C$ or $\ln |\sec u| + C$

1j. $\int \cot u du = \ln |\sin u| + C$ or $-\ln |\csc u| + C$

1k. $\int \sec u du = \ln |\sec u + \tan u| + C$ or $-\ln |\sec u - \tan u| + C$

1l. $\int \csc u du = -\ln |\csc u + \cot u| + C$ or $\ln |\csc u - \cot u| + C$

1m. If a is a constant and $a > 0$ then $\int \frac{1}{\sqrt{a^2 - u^2}} du = \arcsin \frac{u}{a} + C$

1n. If a is a constant and $a > 0$ then $\int \frac{1}{a^2 + u^2} du = \frac{1}{a} \arctan \frac{u}{a} + C$

1o. If a is a constant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2 - a^2}} du = \frac{1}{a} \operatorname{arcsec} \frac{|u|}{a} + C$

1p. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polynomials

and $[\text{degree of } f] \geq [\text{degree of } g]$, then one must first do long division

1q. Integration by parts formula: $\int u dv = uv - \int v du$

1r. Trig substitution: (recall that the *integrand* is the function you are integrating)

if the integrand involves $a^2 - u^2$, then one makes the substitution $u = a \sin \theta$

1s. Trig substitution:

if the integrand involves $a^2 + u^2$, then one makes the substitution $u = a \tan \theta$

1t. Trig substitution:

if the integrand involves $u^2 - a^2$, then one makes the substitution $u = a \sec \theta$

1u. trig formula ... your answer should involve trig functions of θ , and not of 2θ : $\sin(2\theta) = 2 \sin \theta \cos \theta$

1v. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$

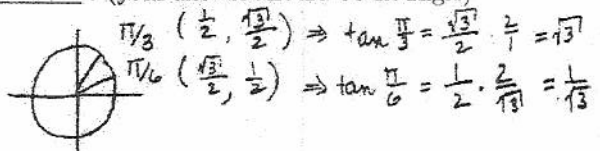
1w. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$

1x. trig formula ... since $\cos^2 \theta + \sin^2 \theta = 1$, we know that the corresponding relationship between

tangent (i.e., \tan) and secant (i.e., \sec) is $1 + \tan^2 \theta = \sec^2 \theta$

1y. $\arctan(-\sqrt{3}) = -\frac{\pi}{3}$ radians. (your answer should be an angle)

$\frac{\cot \theta}{n} \Rightarrow -\frac{\pi}{2} < \arctan \theta < \frac{\pi}{2}$



from 100 Integrals - # 35

$$2. \int (\sec^3 x) (\tan^3 x) dx = \frac{1}{5} \sec^5 x - \frac{1}{3} \sec^3 x + C$$

$$s = \sec x$$
$$ds = \sec x \tan x dx$$

$$t = \tan x$$
$$dt = \sec^2 x dx$$

$$\int \sec^3 x \tan^3 x dx$$

$$= \int \sec^2 x \tan^2 x [\sec x \tan x dx]$$

$$= \int \sec^2 x (\sec^2 x - 1) [\sec x \tan x dx]$$

$$= \int s^2 (s^2 - 1) ds$$

$$= \int (s^4 - s^2) ds$$

$$= \frac{s^5}{5} - \frac{s^3}{3} + C$$

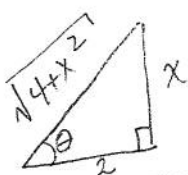
Homework problem § 8.4 # 5

$$3. \int \frac{1}{(4+x^2)^2} dx = \frac{1}{16} \tan^{-1}\left(\frac{x}{2}\right) + \frac{x}{8(4+x^2)} + C$$

$$x = 2 \tan \theta$$

$$dx = 2 \sec^2 \theta d\theta$$

$$4+x^2 = 4 + (2 \tan \theta)^2 = 4 + 4 \tan^2 \theta = 4(1 + \tan^2 \theta) = 4 \sec^2 \theta$$



$$\tan \theta = \frac{x}{2}$$

$$\int \frac{dx}{(4+x^2)^2} = \int \frac{2 \sec^2 \theta d\theta}{(4 \sec^2 \theta)^2} = \frac{2}{4^2} \int \frac{\sec^2 \theta}{\sec^4 \theta} d\theta = \frac{1}{8} \int \frac{d\theta}{\sec^2 \theta}$$

$$= \frac{1}{8} \int \cos^2 \theta d\theta = \frac{1}{8} \cdot \frac{1}{2} \int [1 + \cos 2\theta] d\theta$$

$$= \frac{1}{16} \int d\theta + \frac{1}{16} \cdot \frac{1}{2} \int \cos 2\theta (2 d\theta)$$

$$= \frac{1}{16} \theta + \frac{1}{16} \cdot \frac{1}{2} \cdot \sin 2\theta + C$$

$$= \frac{1}{16} \theta + \frac{1}{16} \cdot \frac{2}{2} \cdot 2 \cdot \cos \theta \cdot \sin \theta + C$$

$$= \frac{1}{16} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{16} \cdot \frac{2}{\sqrt{4+x^2}} \cdot \frac{x}{\sqrt{4+x^2}} + C$$

from 100 Integrals # 71

$$4. \int \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} dx = \frac{1}{2} \ln(x^2 + 1) + \tan^{-1} x + \frac{1}{2} \frac{(x^2 + 1)^{-1}}{-1} + C$$

Hint: $x^4 + 2x^2 + 1 = (x^2 + 1)^2$.

bigger bottoms? yes ✓

$$\frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} = \frac{x^3 + x^2 + 2x + 1}{(x^2 + 1)^2} = \frac{Ax + B}{(x^2 + 1)^1} + \frac{Cx + D}{(x^2 + 1)^2} = \frac{(Ax + B)(x^2 + 1) + Cx + D}{(x^2 + 1)^2}$$

$$\Rightarrow x^3 + x^2 + 2x + 1 = (Ax + B)(x^2 + 1) + Cx + D$$

$$x^3 : 1 = A$$

$$x^2 : 1 = B$$

$$x : 2 = A + C \rightarrow C = 1$$

$$\text{constant} : 1 = B + D \rightarrow D = 0$$

$$\int \frac{x^3 + x^2 + 2x + 1}{x^4 + 2x^2 + 1} dx = \int \left(\frac{1x + 1}{x^2 + 1} + \frac{1x + 0}{(x^2 + 1)^2} \right) dx$$

$$= \frac{1}{2} \int \frac{2x dx}{x^2 + 1} + \int \frac{dx}{x^2 + 1} + \frac{1}{2} \int (x^2 + 1)^{-2} (2x dx)$$

Homework Problem § 8.2 # 27.

5.

$$\int x^3 e^{x^2} dx = \frac{x^2 e^{x^2} - e^{x^2}}{2} + C$$

$$\begin{aligned} u &= \frac{1}{2} x^2 & dv &= e^{x^2} (2x) dx \\ du &= x dx & v &= e^{x^2} \end{aligned}$$

$$\begin{aligned} \int x^3 e^{x^2} dx &= \frac{x^2 e^{x^2}}{2} - \int e^{x^2} x dx \\ &= \frac{x^2 e^{x^2}}{2} - \frac{1}{2} \int e^{x^2} (2x dx) \\ &= \frac{x^2 e^{x^2}}{2} - \frac{1}{2} e^{x^2} + C \end{aligned}$$

Homework Problem § 8.2 # 23

6.
$$\int \sin(\ln x) dx = \frac{1}{2} (x \sin(\ln x) - x \cos(\ln x)) + C$$

Hint: bring to the other side idea.

$$\begin{aligned} u &= \sin(\ln x) & dv &= dx \\ du &= \frac{\cos(\ln x)}{x} dx & v &= x \end{aligned}$$

$$\begin{aligned} \tilde{u} &= \cos(\ln x) & d\tilde{v} &= dx \\ d\tilde{u} &= \frac{-\sin(\ln x)}{x} dx & \tilde{v} &= x \end{aligned}$$

$$\begin{aligned} \int \sin(\ln x) dx &= x \sin(\ln x) - \int \cos(\ln x) dx \\ &= x \sin(\ln x) - [x \cos(\ln x) - \int \sin(\ln x) dx] \\ &= x \sin(\ln x) - x \cos(\ln x) + \int \sin(\ln x) dx \end{aligned}$$

← Hint above

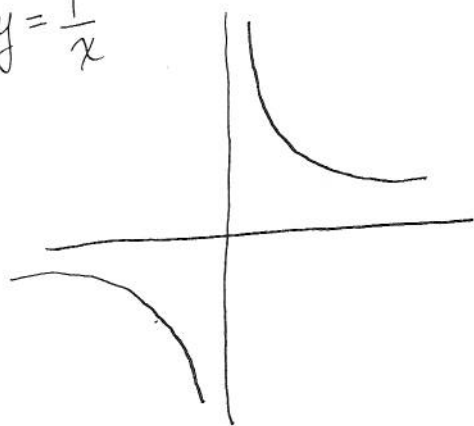
$$2 \int \sin(\ln x) dx = x \sin(\ln x) - x \cos(\ln x) + C$$

7.

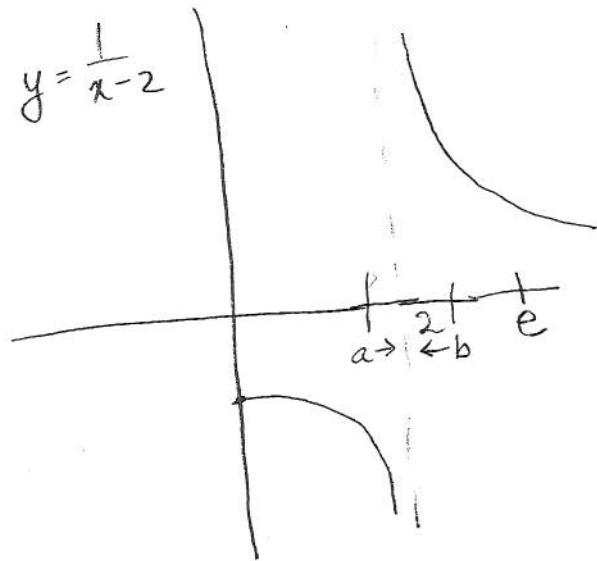
$$\int_0^e \frac{dx}{x-2} = \text{DNE}$$

HINT: $e \approx 2.7 > 2$

$$y = \frac{1}{x}$$


 \Rightarrow

$$y = \frac{1}{x-2}$$



$$\begin{aligned} \int_0^e \frac{dx}{x-2} &= \lim_{a \rightarrow 2^+} \int_0^a \frac{dx}{x-2} + \lim_{b \rightarrow 2^-} \int_b^e \frac{dx}{x-2} \\ &= \lim_{a \rightarrow 2^+} \ln|x-2| \Big|_{x=0}^{x=a} + \lim_{b \rightarrow 2^-} \ln|x-2| \Big|_{x=b}^{x=e} \\ &= \lim_{a \rightarrow 2^+} [\ln|a-2| - \ln 2] + \lim_{b \rightarrow 2^-} [\ln|e-2| - \ln|b-2|] \\ &= \underbrace{\left[\lim_{a \rightarrow 2^+} \ln|a-2| \right]}_{=-\infty} - \ln 2 + \ln|e-2| - \underbrace{\lim_{b \rightarrow 2^-} \ln|b-2|}_{=-\infty} \end{aligned}$$

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TOTAL	10	

NAME (legibly printed): Key

class PIN: 007

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Problem Inspiration: just like the homework.

Honor Code Statement

I understand that it is the responsibility of every member of the Carolina community to uphold and maintain the University of South Carolina's Honor Code.

As a Carolinian, I certify that I have neither given nor received unauthorized aid on this exam.

Furthermore, I have not only read but will also follow the above Instructions.

I hereby verify that I did NOT receive help from other people on this take-home exam problem.

Signature : James Bond

7. Use inequality (11), page 563, to find an upper bound on the error in part 3. Do not use a calculator.

$$|T_6 - I| \leq \frac{(4-1)^3}{12(6)^2} \cdot \frac{6}{2^4} \text{ or } \frac{3}{2^7} \text{ or } \frac{3}{128}$$

$$K_2 = \max_{1 \leq x \leq 4} |f^{(2)}(x)| = \max_{1 \leq x \leq 4} \frac{6}{(x+1)^4} = \frac{6}{2^4}$$

$$\Rightarrow 1 \leq x \leq 4 \Rightarrow 2 \leq x+1 \leq 5 \Rightarrow \frac{1}{5} \leq \frac{1}{x+1} \leq \frac{1}{2}$$

8. Use your calculator to approximate the error estimate in part 7 to as many decimal places as your calculator will give you.

$$|T_6 - I| \approx 0.0234375$$

9. Use inequality (12), page 563 to find an upper bound on the error in part 5. Do not use a calculator.

$$|S_6 - I| \leq \frac{(4-1)^5}{180(6)^4} \cdot \frac{120}{2^6} \text{ or } \frac{1}{2^9}$$

$$K_4 = \max_{1 \leq x \leq 4} |f^{(4)}(x)| = \max_{1 \leq x \leq 4} \frac{120}{(x+1)^6} = \frac{120}{2^6}$$

10. Use your calculator to approximate the error estimate in part 9, to as many decimal places as your calculator will give you.

$$|S_6 - I| \approx 0.001953125$$

5. Approximate I using Simpson's Rule S_{2n} with $2n = 6$ subintervals. Do not use a calculator (so your answer will have several numbers added together).

$$S_6 = \left(\frac{1}{3}\right) \left(\frac{4-1}{6}\right) \left[\frac{1}{2^2} + \frac{4}{(2.5)^2} + \frac{2}{3^2} + \frac{4}{(3.5)^2} + \frac{2}{4^2} + \frac{4}{(4.5)^2} + \frac{1}{5^2} \right]$$

6. Find an approximation for S_6 , using part 5 and your calculator, to as many decimal places as your calculator will give you.

$$S_6 \approx 0.3002139498$$

- *. Find the first 4 derivatives of the given $y = f(x)$ that we are working with.

$$f(x) = (x+1)^{-2}$$

$$f^{(1)}(x) = -2(x+1)^{-3}$$

$$f^{(2)}(x) = 6(x+1)^{-4}$$

$$f^{(3)}(x) = -24(x+1)^{-5}$$

$$f^{(4)}(x) = 120(x+1)^{-6}$$

Numerical Integration . Let

$$f(x) = \frac{1}{(x+1)^2} \quad \text{and} \quad I = \int_{x=1}^{x=4} f(x) dx .$$

The 10 steps of this problem are similar to the homework but the number of subintervals is 6 and not 10. On the parts that say "Do not use a calculator", you need not do alot of arthritic.

1. Find the exact value of I , without using a calculator. Your answer should be a fraction, without decimal places.

$$I = \frac{3}{10}$$

$$\int_{x=1}^{x=4} (x+1)^{-2} dx = \frac{(x+1)^{-1}}{-1} \Big|_{x=1}^{x=4} = \frac{1}{x+1} \Big|_{x=1}^{x=4} = \frac{1}{2} - \frac{1}{5}$$

$$= \frac{5-2}{10} = \frac{3}{10}$$

2. Find an approximation for I , using part 1 and your calculator, to as many decimal places as your calculator will give you.

$$I \approx .3$$

3. Approximate I using the Trapezoid Rule T_n with $n = 6$ subintervals. Do not use a calculator (so your answer will have several numbers added together).

$$T_6 = \frac{4-1}{2(6)} \left[\frac{1}{2^2} + \frac{2}{(2.5)^2} + \frac{2}{3^2} + \frac{2}{(3.5)^2} + \frac{2}{4^2} + \frac{2}{(4.5)^2} + \frac{1}{5^2} \right]$$

Divide $[1, 4]$ into 6 subintervals $\rightarrow \Delta x = \frac{b-a}{n} = \frac{4-1}{6} = \frac{3}{6} = \frac{1}{2} = .5$

x_0	x_1	x_2	x_3	x_4	x_5	x_6
$a = 1$	$1\frac{1}{2} = 1.5 = \frac{3}{2}$	2	$2\frac{1}{2} = 2.5 = \frac{5}{2}$	3	$3\frac{1}{2} = 3.5 = \frac{7}{2}$	$4 = b$

4. Find an approximation for T_6 , using part 3 and your calculator, to as many decimal places as your calculator will give you.

$$T_6 \approx 0.3048132401$$