

| MARK BOX |        |  |
|----------|--------|--|
| PROBLEM  | POINTS |  |
| 1 a - o  | 15     |  |
| 2        | 12     |  |
| 3        | 12     |  |
| 4        | 12     |  |
| 5        | 12     |  |
| 6        | 12     |  |
| 7        | 12     |  |
| 8        | 13     |  |
| %        | 100    |  |

NAME: \_\_\_\_\_

please check the box of your section

Section 001 (MW 9:05 am)

or

Section 002 (MW 10:10 am)

**INSTRUCTIONS:**

- (1) To receive credit you must:
  - (a) **work in a logical fashion, show all your work, indicate your reasoning;**  
no credit will be given for an answer that *just appears*;  
such explanations help with partial credit
  - (b) if a line/box is provided, then:
    - show you work BELOW the line/box
    - put your answer on/in the line/box
  - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.  
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8<sup>th</sup> ed.):  
the whole of Chapter 10: Sections 10.1 - 10.10 .

**Problem Inspiration:**

1. From class handouts. You were warned.
2. homework problem § 10.1 # 15 and Mo's Friday homework # 2
3. Serious Series Problems # 4
4. Mo's Friday homework # 2
5. homework problem § 10.8 # 35
6. Example from class.
7. Mo's Friday homework # 2
8. homework problem § 10.9 # 3

1. Fill-in-the blanks/boxes. All series  $\sum$  are understood to be  $\sum_{n=1}^{\infty}$ .

**Hint: I do NOT want to see the words absolute nor conditional on this page!**

**1a. Sequences** Let  $-\infty < r < \infty$ . (Fill-in-the blanks with *exists* or *does not exist*, i.e. *DNE*)

- If  $|r| < 1$ , then  $\lim_{n \rightarrow \infty} r^n$  \_\_\_\_\_
- If  $|r| > 1$ , then  $\lim_{n \rightarrow \infty} r^n$  \_\_\_\_\_
- If  $r = 1$ , then  $\lim_{n \rightarrow \infty} r^n$  \_\_\_\_\_
- If  $r = -1$ , then  $\lim_{n \rightarrow \infty} r^n$  \_\_\_\_\_

**1b. Geometric Series** where  $-\infty < r < \infty$ . The series  $\sum r^n$

- converges if and only if  $|r|$  \_\_\_\_\_
- diverges if and only if  $|r|$  \_\_\_\_\_

**1c.  $p$ -series** where  $0 < p < \infty$ . The series  $\sum \frac{1}{n^p}$

- converges if and only if  $p$  \_\_\_\_\_
- diverges if and only if  $p$  \_\_\_\_\_

**1d. Integral Test** for a positive-termed series  $\sum a_n$  where  $a_n \geq 0$ .

Let  $f: [1, \infty) \rightarrow \mathbb{R}$  be so that

- $a_n = f(\text{_____})$  for each  $n \in \mathbb{N}$
- $f$  is a \_\_\_\_\_ function
- $f$  is a \_\_\_\_\_ function
- $f$  is a \_\_\_\_\_ function .

Then  $\sum a_n$  converges if and only if \_\_\_\_\_ converges.

**1e. Comparison Test** for a positive-termed series  $\sum a_n$  where  $a_n \geq 0$ .

- If  $0 \leq a_n \leq b_n$  for all  $n \in \mathbb{N}$  and  $\sum b_n$  \_\_\_\_\_, then  $\sum a_n$  \_\_\_\_\_.
- If  $0 \leq b_n \leq a_n$  for all  $n \in \mathbb{N}$  and  $\sum b_n$  \_\_\_\_\_, then  $\sum a_n$  \_\_\_\_\_.

**1f. Limit Comparison Test** for a positive-termed series  $\sum a_n$  where  $a_n \geq 0$ .

Let  $b_n > 0$  and  $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$ .

If \_\_\_\_\_  $< L <$  \_\_\_\_\_, then  $\sum a_n$  converges if and only if \_\_\_\_\_ .

**1g. Ratio and Root Tests** for a positive-termed series  $\sum a_n$  where  $a_n \geq 0$ .

Let  $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$  or  $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$ .

- If  $\rho$  \_\_\_\_\_ then  $\sum a_n$  converges.
- If  $\rho$  \_\_\_\_\_ then  $\sum a_n$  diverges.
- If  $\rho$  \_\_\_\_\_ then the test is inconclusive.

**1h. Alternating Series Test** for an alternating series  $\sum (-1)^n a_n$  where  $a_n > 0$  for each  $n \in \mathbb{N}$ .

If

- $a_n$  \_\_\_\_\_  $a_{n+1}$  for each  $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n =$  \_\_\_\_\_

then  $\sum (-1)^n a_n$  \_\_\_\_\_

**1i.  $n^{\text{th}}$ -term test** for an arbitrary series  $\sum a_n$ .

If  $\lim_{n \rightarrow \infty} a_n \neq 0$  or  $\lim_{n \rightarrow \infty} a_n$  does not exist, then  $\sum a_n$  \_\_\_\_\_ .



2. For the following **SEQUENCES**:

- if the limit exists, find it
- if the limit does not exist, then say that it DNE.

Put your ANSWER IN the box and show your WORK BELOW the box.

2a.

$$\lim_{n \rightarrow \infty} \frac{(3n + 1)(5n + 2)}{17n^2} =$$

2b.

$$\lim_{n \rightarrow \infty} (-1)^n \frac{(3n + 1)(5n + 2)}{17n^2} =$$

2c.

$$\lim_{n \rightarrow \infty} (1.00000017)^n =$$

3. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n^8 + 1}}$$

- absolutely convergent
- conditionally convergent
- divergent

4. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=8}^{\infty} (-1)^n \frac{1}{(\ln n)^3}$$

absolutely convergent

conditionally convergent

divergent

Hint: For any  $0 < q < \infty$ , if  $n$  is big enough then  $\ln n < n^q$  and so  $\frac{1}{(n^q)^3} < \frac{1}{(\ln n)^3}$ .

Hint: the integral test is NOT helpful.

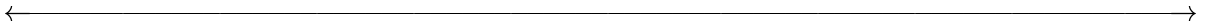
5. Consider the formal power series

$$\sum_{n=1}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

Hint:  $\left| \frac{x^{2(n+1)+1}}{(2(n+1)+1)!} \frac{(2n+1)!}{x^{2n+1}} \right| = \frac{|x|^{2n+3}}{|x|^{2n+1}} \frac{(2n+1)!}{(2n+3)!} = \frac{|x|^2}{1} \frac{(2n+1)!}{(2n+1)! (2n+2) (2n+3)} .$

The center is  $x_0 =$  \_\_\_\_\_ and the radius of convergence is  $R =$  \_\_\_\_\_ .

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



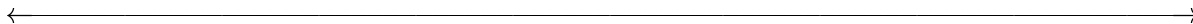
6. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(3x - 6)^n}{n}$$

Hint:  $(3x - 6)^n = [3(x - 2)]^n = 3^n (x - 2)^n$ .

The center is  $x_0 =$  \_\_\_\_\_ and the radius of convergence is  $R =$  \_\_\_\_\_.

As we did in class, make a number line indicating where the power series is: absolutely convergent, conditionally convergent, and divergent. Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.



7. Using the fact that

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n \quad \text{when} \quad |r| < 1, \quad (*)$$

find a power series expansion of

$$\frac{x}{2 + 32x^4}$$

and state when it is valid. Simplify your answer so that your power series has the form

$\sum_{n=0}^{\infty} c_n x^{\text{some power}}$  for some constants  $c_n$ .

$$\frac{x}{2 + 32x^4} = \sum_{n=0}^{\infty} \quad \text{when } |x| <$$

Hint:  $32 = 2(16)$  and  $16 = 2^4$ .

8. In this problem, you may **NOT** use a known Taylor series; instead, you must compute requested items by hand. Let

$$f(x) = e^{-x} \quad \text{and} \quad x_0 = 17.$$

We will follow the notation from this exam on **PAGE 3** problems **11 - 1o**, so here

- $y = p_\infty(x)$  is the Taylor series of  $y = e^{-x}$  about  $x_0 = 17$
- $y = p_N(x)$  is the  $N^{\text{th}}$ -order Taylor polynomial of  $y = e^{-x}$  about  $x_0 = 17$
- $y = R_N(x)$  is the  $N^{\text{th}}$ -order Taylor remainder of  $y = e^{-x}$  about  $x_0 = 17$ .

You may use, without showing, that

$$f^{(n)}(x) = (-1)^n e^{-x}$$

for  $n = 0, 1, 2, 3, 4, \dots$ .

- 8a. In closed form (i.e., with a  $\sum$ -sign and without ...)

$p_\infty(x) =$

Your answer should NOT have the symbol  $f^{(n)}$  in it.

- 8b. We know that  $f(16) = p_N(16) + R_N(16)$ . Taylor's BIG Theorem tells us that, for  $x = 16$ ,

$R_N(16) =$

for some  $c$  between

and

.

Your answer should have a  $N$  and a  $c$  in it but it should NOT have a  $x$  nor  $x_0$  in it.

- 8c. Find an upper bound for  $|R_N(16)|$ .

$|R_N(16)| \leq$

Your answer should have an  $N$  in it but should NOT have a  $c$  nor  $x$  nor  $x_0$  in it.

- 8d. Show that  $f(16) = p_\infty(16)$ .