

## Math 142

In-Class Quiz from Thursday 2/9/06

Due at beginning of class on Monday 2/13/06

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The class performance on Problem 1 from Exam 1 was **absolutely unacceptable**: the average score was only 5.6 and the median was only 5 (out of 10 points). You were told that the first problem would be fill-in the blanks from the formulas on the **Math 141 Handout** that I handed out the first day of class. Thus here is a quiz to reinforce some of formulas.

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Using only

- (i) simple  $u - du$  substitution
- (ii) basic trig identities such as

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x} \quad \sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x} \quad (\text{A})$$

- (iii) the fact that  $y = e^x$  and  $y = \ln x$  are inverse functions of each other and so

$$a^x = e^{\ln(a^x)} = e^{(x \ln a)} \quad (\text{B})$$

where  $a$  is a constant with  $a > 0$  and  $a \neq 1$

- (iv) If  $z > 0$  then

$$\ln \frac{1}{z} = \ln 1 - \ln z = -\ln z \quad (\text{C})$$

derive the formulas listed in (1) through (5) below. These formulas are from your Math 141 Handout. Derive means to show that they *easily* follow from (i) through (iv) above.

**So if you ever forget the below formulas on an exam again, you will know how to easily and quickly derive them.**

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- (1) If  $a$  is a constant and  $a > 0$  but  $a \neq 1$ , then

$$\int a^x dx = \frac{a^x}{\ln a} + C .$$

Hint: use (B) above (more than once) and let  $u = x \ln a$ .

(2)

$$\int \tan x \, dx = -\ln |\cos x| + C$$
$$\int \tan x \, dx = \ln |\sec x| + C$$

Hints: use (A) and (C) above and let  $u = \cos x$ .

(3)

$$\int \cot x \, dx = \ln |\sin x| + C$$
$$\int \cot x \, dx = -\ln |\csc x| + C$$

Hints: use (A) and (C) above and let  $u = \sin x$ .

(4)

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$
$$\int \sec x \, dx = -\ln |\sec x - \tan x| + C$$
$$\int \sec x \, dx = -\ln |\tan x - \sec x| + C$$

Hints.

First multiply the integrand through by the number one, written in a clever way:

$$\frac{\sec x + \tan x}{\sec x + \tan x} \quad \text{or} \quad \frac{\sec x - \tan x}{\sec x - \tan x} \quad \text{or} \quad \frac{\tan x - \sec x}{\tan x - \sec x} .$$

Then let  $u$  be either:

$$\sec x + \tan x \quad \text{or} \quad \sec x - \tan x \quad \text{or} \quad \tan x - \sec x .$$

(5)

$$\int \csc x \, dx = \ln |\csc x - \cot x| + C$$
$$\int \csc x \, dx = \ln |\cot x - \csc x| + C$$
$$\int \csc x \, dx = -\ln |\csc x + \cot x| + C$$

Hints.

First multiply the integrand through by the number one, written in a clever way:

$$\frac{\csc x - \cot x}{\csc x - \cot x} \quad \text{or} \quad \frac{\cot x - \csc x}{\cot x - \csc x} \quad \text{or} \quad \frac{\csc x + \cot x}{\csc x + \cot x} .$$

Then let  $u$  be either:

$$\csc x - \cot x \quad \text{or} \quad \cot x - \csc x \quad \text{or} \quad \csc x + \cot x .$$