MARK BOX		
PROBLEM	POINTS	
1 a – y	25	
2 a – o	15	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17 a – f	10	
18	10	
TOTAL	200	

NAME: ______________________________

please check the box of your section

☐ **Section 005 (WF 8:00 am)**

or

☐ **Section 006 (WF 9:05 am)**

Problem Inspiration:
1.–3. Fill in blanks & True/False
4.–5. Ch. 7
6.–11. Ch. 8
12.–16. § 10.1 – 10.6 and 10.8
17.–18. § 10.7, 10.9, 10.10

INSTRUCTIONS:

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning**; **no credit will be given for an answer that *just appears***; such explanations help with partial credit
  (b) if a line/box is provided, then:
  — show you work BELOW the line/box
  — put your answer on/in the line/box
  (c) if no such line/box is provided, then box your answer
(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
(3) You may **not** use a calculator, books, personal notes.
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8^{th} ed.): Sections: 7.1 - 7.4, 7,6, 7.7, 8.1 - 8.5, 8.8, 10.1 - 10.10 .

1. Fill in the blanks.

1a. $\int \frac{du}{u} \;=$ __________ $|u| + C$

1b. If a is a constant and $a > 0$ but $a \neq 1$, then $\int a^u \, du \;=$ ______________________________ $+C$

1c. $\int \cos u \, du \;=$ ____ $+C$

1d. $\int \sec^2 u \, du \;=$ ____ $+C$

1e. $\int \sec u \tan u \, du \;=$ ____ $+C$

1f. $\int \sin u \, du \;=$ ____ $+C$

1g. $\int \csc^2 u \, du \;=$ ____ $+C$

1h. $\int \csc u \cot u \, du \;=$ ____ $+C$

1i. $\int \tan u \, du \;=$ ____ $+C$

1j. $\int \cot u \, du \;=$ ____ $+C$

1k. $\int \sec u \, du \;=$ ____ $+C$

1l. $\int \csc u \, du \;=$ ____ $+C$

1m. If a is a contant and $a > 0$ then $\int \frac{1}{\sqrt{a^2-u^2}} \, du \;=$ __________________________________ $+C$

1n. If a is a contant and $a > 0$ then $\int \frac{1}{a^2+u^2} \, du \;=$ __________________________________ $+C$

1o. If a is a contant and $a > 0$ then $\int \frac{1}{u\sqrt{u^2-a^2}} \, du \;=$ __________________________________ $+C$

1p. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polyonomials and [degree of f] $\geq$ [degree of g], then one must first do ______________________________

1q. Integration by parts formula: $\int u \, dv \;=$ ____

1r. Trig substitution: (recall that the *integrand* is the function you are integrating)
if the integrand involves a^2-u^2, then one makes the substitution $u \;=$ ________________________

1s. Trig substitution:
if the integrand involves a^2+u^2, then one makes the substitution $u \;=$ ________________________

1t. Trig substitution:
if the integrand involves u^2-a^2, then one makes the substitution $u \;=$ ________________________

1u. trig formula ... your answer should involve trig functions of θ, and not of 2θ: $\sin(2\theta) =$ ________________ .

1v. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\cos^2(\theta) \;= \frac{\qquad\qquad\qquad}{2}$.

1w. trig formula ... $\cos(2\theta)$ should appear in the numerator: $\sin^2(\theta) \;= \frac{\qquad\qquad\qquad}{2}$.

1x. trig formula ... since $\cos^2\theta + \sin^2\theta = 1$, we know that the corresponding relationship beween tangent (i.e., tan) and secant (i.e., sec) is ____ .

1y. $\arctan(-\sqrt{3}\;) =$ __________________ **RADIANS**, not degrees.

2. Fill-in-the blanks/boxes. All series $\sum$ are understood to be $\sum_{n=1}^{\infty}$.

Hint: I do NOT want to see the words absolute nor conditional on this page!

2a. Sequences Let $-\infty < r < \infty$. (Fill-in-the blanks with *exists* or *does not exist, i.e. DNE*)

- If $|r| < 1$, then $\lim_{n\to\infty} r^n$ __________________________
- If $|r| > 1$, then $\lim_{n\to\infty} r^n$ __________________________
- If $r = 1$, then $\lim_{n\to\infty} r^n$ __________________________
- If $r =$ -1, then $\lim_{n\to\infty} r^n$ __________________________

2b. Geometric Series where $-\infty < r < \infty$. The series $\sum r^n$

- converges if and only if $|r|$ __________________
- diverges if and only if $|r|$ __________________

2c. p**-series** where $0 < p < \infty$. The series $\sum \frac{1}{n^p}$

- converges if and only if p __________________
- diverges if and only if p __________________

2d. Integral Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $f\colon [1,\infty) \to \mathbb{R}$ be so that

- $a_n = f($_________$)$ for each $n \in \mathbb{N}$
- f is a ____________________________________ function
- f is a ____________________________________ function
- f is a ____________________________________ function .

Then $\sum a_n$ converges if and only if ____________________________________ converges.

2e. Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ __________________, then $\sum a_n$ __________________.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n$ __________________, then $\sum a_n$ __________________.

2f. Limit Comparison Test for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $b_n > 0$ and $\lim_{n\to\infty} \frac{a_n}{b_n} = L$.

If _________ $< L <$ _________, then $\sum a_n$ converges if and only if __________________ .

2g. Ratio and Root Tests for a positive-termed series $\sum a_n$ where $a_n \geq 0$.

Let $\rho = \lim_{n\to\infty} \frac{a_{n+1}}{a_n}$ or $\rho = \lim_{n\to\infty} (a_n)^{\frac{1}{n}}$.

- If ρ _________ then $\sum a_n$ converges.
- If ρ _________ then $\sum a_n$ diverges.
- If ρ _________ then the test is inconclusive.

2h. Alternating Series Test for an alternating series $\sum (-1)^n a_n$ where $a_n > 0$ for each $n \in \mathbb{N}$.

If

- a_n _________ a_{n+1} for each $n \in \mathbb{N}$
- $\lim_{n\to\infty} a_n =$ _________

then $\sum (-1)^n a_n$ __________________________

2i. n^{th}**-term test** for an arbitrary series $\sum a_n$.

If $\lim_{n\to\infty} a_n \neq 0$ or $\lim_{n\to\infty} a_n$ does not exist, then $\sum a_n$ __________________ .

2j. By definition, for an arbitrary series $\sum a_n$, (fill in the blanks with converges or diverges).

- $\sum a_n$ is absolutely convergent if and only if $\sum |a_n|$ ____________________
- $\sum a_n$ is conditionally convergent if and only if $\sum a_n$ __________ and $\sum |a_n|$ __________
- $\sum a_n$ is divergent if and only if $\sum a_n$ ____________________

2k. Fill in the 3 blank lines, with

absolutely convergent, conditional convergent, or divergent,

on the following FLOW CHART for class used to determine if a series $\sum_{n=17}^{\infty} a_n$ is: absolutely convergent, conditional convergent, or divergent.

Does $\sum |a_n|$ converge?
Since $|a_n| \geq 0$, use a positive term test: integral test, CT, LCT, ratio/root test.

if NO $\Rightarrow$ Does $\lim_{n\to\infty} |a_n| = 0$?

if NO $\Rightarrow$ $\sum a_n$ is ________________

(first box) if YES $\Downarrow$

$\sum a_n$ is ______________________________

(second box) if YES $\Downarrow$

Is $\sum a_n$ an alternating series?

if YES $\Downarrow$

Does $\sum a_n$ satisfy the conditions of the Alternating Series Test?

if YES $\Downarrow$

$\sum a_n$ is ______________________________

2l. Fill in the blank with: **perpendicular or parallel**.

If you revolve a graph of a function about an axis of revolution and you want to find the volume of the resulting solid of revolution using the **disk or washer method**, then you draw in your typical rectangles (used to form your typical elements) ______________________ to the axis of revolution.

2m. Fill in the blank with a formula involving *some of*:

2 , π , radius , $\text{radius}_{\text{big}}$, $\text{radius}_{\text{little}}$, average radius , height , and/or thickness.

In problem **2l** above, if you use the **washer method**, then the volume of a typical washer is:

____ .

2n. Fill in the blank with: **perpendicular or parallel**.

If you revolve a graph of a function about an axis of revolution and you want to find the volume of the resulting solid of revolution using the **shell method**, then you draw in your typical rectangles (used to form your typical elements) ______________________ to the axis of revolution.

2o. Fill in the blank with a formula involving *some of*:

2 , π , radius , $\text{radius}_{\text{big}}$, $\text{radius}_{\text{little}}$, average radius , height , and/or thickness.

In problem **2n** above, if you use the **shell method**, then the volume of a typical shell is:

____ .

3. Circle T if the statement is (always) TRUE. Circle F if the statement is (sometimes) FALSE.

T F If $f(n) = a_n$ for $n = 1, 2, 3, \ldots$ and if $\lim_{x\to\infty} f(x) = L$ then $\lim_{n\to\infty} a_n = L$.

T F If $f(n) = a_n$ for $n = 1, 2, 3, \ldots$ and if $\lim_{n\to\infty} a_n = L$, then $\lim_{x\to\infty} f(x) = L$.

T F If $0 < a_n < 1$, then $\lim_{n\to\infty} a_n$ exists.

T F If $0 < a_n < 1$, then $\sum a_n$ exists.

T F If $\lim_{n\to\infty} a_n = 0$, then $\sum a_n$ converges

T F If $\sum a_n$ converges, then $\lim_{n\to\infty} a_n = 0$.

T F If $\sum(a_n + b_n)$ converges, then $\sum a_n$ converges and $\sum b_n$ converge.

T F If $\sum a_n$ converges and $\sum b_n$ converge, then $\sum(a_n + b_n)$ converges.

T F If $S_N = \displaystyle\sum_{n=2}^{N-1} r^n$, then $S_N = \dfrac{r^2 - r^N}{1 - r}$. Notice that the sum is from **2** to **N-1**

T F Calculus is fun! (each **honest** solution is correct!)

For problem 4 and 5: Let R be the region in the first quadrant of the xy-plane enclosed by:

- the parabola $y = 3x^2$
- the x-axis
- the vertical line $x = 2$.

Let V be the volume of the solid obtained by revolving the region R about the $\boxed{y\text{-axis}}$.

Please remind Prof. Girardi to sketch R on the board for you.

4. Using the **disk or washer method**, express the volume V as an integral.

V =

You do not have to evaluate the integral nor do lot of algebra to the integrand.

5. Using the **shell method**, express the volume V as an integral.

V =

You do not have to evaluate the integral nor do lot of algebra to the integrand.

6. $$\int \sec^4 x \tan^4 x \; dx \;\; = \qquad\qquad\qquad\qquad +C$$

Hint: problem 1x.

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

7.

$$\int \frac{dx}{(4+x^2)^2} \; = \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad + C$$

Hint: problem 1s, 1u, 1v, 1w.

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

8. $\displaystyle\int \frac{2x^2 - 2x - 1}{x^2\ (x-1)}\ dx \ =$ $+C$

Hint: $x^2 \ = \ (x-0)^2 \ = \ (\text{a linear term})^2 \ \neq \ (\text{an irreducible quadratic})^1$.

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

9. $\displaystyle\int x^2\ e^x\ dx\ =$ $+C$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

10. $$\int \ln(x+17)\,dx \;=\; \qquad\qquad\qquad +C$$

Hint: some cleverness in your choice of v can make life easier.

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

11. $\int e^{(2x)} \cos(3x)\, dx \;=$ $+C$

Hint: bring to the other side idea.

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

12. Sequences (<u>not series</u>)

12a. $$\lim_{n\to\infty} \frac{\ln(n^2)}{n} =$$

12b. $$\lim_{n\to\infty} \frac{\sqrt{4n^3+7n+5}}{7n^{\frac{3}{2}}+8} =$$

NOW Series - NOT sequences

13. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=17}^{\infty} \frac{(-1)^n}{n}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

14. Check the correct box and then indicate your reasoning below. Specifically specify what test(s) you are using. A correctly checked box without appropriate explanation will receive no points.

$$\sum_{n=17}^{\infty} \frac{1}{\sqrt{(n-1)\,(n-2)\,(n-3)}}$$

☐ absolutely convergent

☐ conditionally convergent

☐ divergent

15. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x-12)^n}{n}$$

Hint: $(2x-12)^n \;=\; [2\,(x-6)]^n \;=\; 2^n\,(x-6)^n$.

The center is $x_0 =$ __________________ and the radius of convergence is $R =$ __________________ .

As we did in class, make a number line indicating where the power series is:

absolutely convergent, conditionally convergent, and divergent.

Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

16. Consider the formal power series

$$\sum_{n=2}^{\infty} \frac{(n!)\ x^n}{2^n}\ .$$

The center is $x_0 =$ __________________ and the radius of convergence is $R =$ __________________ .
As we did in class, make a number line indicating where the power series is:

absolutely convergent, conditionally convergent, and divergent.

Indicate your reasoning and specifically specify what test(s) you are using. Don't forget to check the endpoints, if there are any.

17. Do parts (a) - (f) for the following:

$$f(x) \ = \ xe^x \qquad x_0 \ = \ 0 \qquad J \ = \ (\text{-}17\ ,\ 2)$$

You might find it easier to do problems (a) - (f) in a different order. Just do what you find easiest.

Use only:

- the definition of Taylor polynominal
- the definition of Taylor series
- the theorem/error-estimate on the N^{th}-Remainder term for Taylor polynomials.

Do **NOT** use a known Taylor Series (i.e., do not use methods from section 10.10).

17a. Find the following. Note the 1st column are functions of x and the 2nd and 3rd columns are numbers

$f^{(0)}(x) \ =$	$f^{(0)}(x_0) \ =$	$c_0 \ =$
$f^{(1)}(x) \ =$	$f^{(1)}(x_0) \ =$	$c_1 \ =$
$f^{(2)}(x) \ =$	$f^{(2)}(x_0) \ =$	$c_2 \ =$
$f^{(3)}(x) \ =$	$f^{(3)}(x_0) \ =$	$c_3 \ =$
$f^{(4)}(x) \ =$	$f^{(4)}(x_0) \ =$	$c_4 \ =$
$f^{(5)}(x) \ =$	nothing for here	nothing for here

17b. Find the N^{th}-order Taylor polynomial of $y = f(x)$ about x_0 in OPEN form for $N = 0, 1, 2, 3, 4$.

$P_0(x) \ =$
$P_1(x) \ =$
$P_2(x) \ =$
$P_3(x) \ =$
$P_4(x) \ =$

17c. Find the Taylor series of $y = f(x)$ about x_0 in OPEN form.

$P_\infty(x) \ =$

17d Find the Taylor series of $y = f(x)$ about x_0 in CLOSED form.

$P_\infty(x) \ =$

17e. Consider the given interval J. Find an upper bound for the maximum of $\left|f^{(5)}(x)\right|$ on the interval J. You answer should be a number. You answer cannot have an: N, x, x_0, c.

$\max\limits_{c \in J} \left|f^{(5)}(c)\right| \ \leq$

17f. Consider the given interval J. Using Taylor's Remainder Theorem (i.e., Taylor's Big Theorem), find an upper bound for the maximum of $|R_4(x)|$ on the interval J. You answer should be a number. You answer cannot have an: N, x, x_0, c.

$\max\limits_{x \in J} |R_4(x)| \ \leq$

18. Using the fact that

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n \qquad \text{when} \qquad |r| < 1\ , \qquad (*)$$

find a power series expansion of

$$\frac{x}{4+100x^2}$$

and state when it is valid. Simplify your answer so that your power series has the form $\sum_{n=0}^{\infty} c_n x^{\text{some power}}$ for some constants c_n.

$\frac{x}{4+100x^2} = \sum_{n=0}^{\infty}$ valid when $\lvert x \rvert <$