

MARK BOX		
PROBLEM	POINTS	
1 a – y	25	in class
2 a – o	15	in class
3	10	in class
4 – 16 (10 pts each)	130	in class
17 a–f <b>take home</b>	10	
18 <b>take home</b>	10	
TOTAL	200	

NAME: \_\_\_\_\_

**please check the box of your section**

**Section 005 (WF 8:00 am)**

or

**Section 006 (WF 9:05 am)**

The (whole) final exam covers (from *Calculus* by Anton, Bivens, Davis 8<sup>th</sup> ed.):  
 Sections: 7.1 - 7.4, 7.6, 7.7, 8.1 - 8.5, 8.8, 10.1 - 10.10 .

The above **MARK BOX** and below **Problem Inspiration** gives you an idea of the break down.

**Problem Inspiration:**

**IN CLASS: 1.–16.**

**1.–3.** Fill in the blanks and True/False (omitting sections 10.7, 10.9, 10.10)

**4.&5.** Chapter 7

**6.–11.** Chapter 8

**12.–16** Sections 10.1 – 10.6 and 10.8

**TAKE HOME 17.–18.**

**17.&18** Sections 10.7, 10.9, 10.10

**INSTRUCTIONS for TAKE HOME PART**, which is due at 2pm on December 15, 2006:

- (1) **Turn in all 5 pages of this exam.**
- (2) To receive credit you must:
  - (a) **work in a logical fashion, show all your work, indicate your reasoning;**  
**no credit will be given for an answer that *just appears*;**  
such explanations help with partial credit
  - (b) if a line/box is provided, then:
    - show you work **BELOW** the line/box
    - put your answer on/in the line/box
  - (c) if no such line/box is provided, then box your answer
- (3) You can use books and notes.
- (4) **You can not use a calculator. You can not use a computer.** Thus you do not need to do lots of *multiplication* and may leave you answers as you would on previous exams (e.g.  $\frac{(7)(17)}{51}$  is acceptable).
- (5) **You can not receive help from other people.**

**SIGNATURE REQUIRED:**

I hereby verify that I did NOT receive help from other people on this final exam take-home part.

Signature: \_\_\_\_\_

---

Nothing to fill in on this page – Prof. Girardi just included it to help you out.

---

Let  $y = f(x)$  be a function with derivatives of all orders in an interval  $I$  containing  $x_0$ .

Let  $y = P_N(x)$  be the  $N^{\text{th}}$ -order Taylor polynomial of  $y = f(x)$  about  $x_0$ .

Let  $y = R_N(x)$  be the  $N^{\text{th}}$ -order Taylor remainder of  $y = f(x)$  about  $x_0$ .

Let  $y = P_\infty(x)$  be the Taylor series of  $y = f(x)$  about  $x_0$ .

Let  $c_n$  be the  $n^{\text{th}}$  Taylor coefficient of  $y = f(x)$  about  $x_0$ .

**A.** In open form (i.e., with  $\dots$  and without a  $\sum$ -sign)

$$P_N(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \frac{f^{(3)}(x_0)}{3!}(x - x_0)^3 + \dots + \frac{f^{(N)}(x_0)}{N!}(x - x_0)^N$$

**B.** In closed form (i.e., with a  $\sum$ -sign and without  $\dots$ )

$$P_N(x) = \sum_{n=0}^N \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

**C.** In open form (i.e., with  $\dots$  and without a  $\sum$ -sign)

$$P_\infty(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f^{(2)}(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \dots$$

**D.** In closed form (i.e., with a  $\sum$ -sign and without  $\dots$ )

$$P_\infty(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} (x - x_0)^n$$

**E.** We know that  $f(x) = P_N(x) + R_N(x)$ . Taylor's BIG Theorem tells us that, for each  $x \in I$ ,

$$R_N(x) = \frac{f^{(N+1)}(c)}{(N+1)!} (x - x_0)^{(N+1)} \quad \text{for some } c \text{ between } \boxed{x} \text{ and } \boxed{x_0} .$$

**F.** The formula for  $c_n$  is

$$c_n = \frac{f^{(n)}(x_0)}{n!}$$

17. Do parts (a) - (f) for the following:

$$f(x) = xe^x \quad x_0 = 0 \quad J = (-17, 2)$$

You might find it easier to do problems (a) - (f) in a different order. Just do what you find easiest.

Use only:

- the definition of Taylor polynomial
- the definition of Taylor series
- the theorem/error-estimate on the  $N^{\text{th}}$ -Remainder term for Taylor polynomials.

**Do NOT use a known Taylor Series (i.e., do not use methods from section 10.10).**

17a Find the following. Note the 1<sup>st</sup> column are functions of  $x$  and the 2<sup>nd</sup> and 3<sup>rd</sup> columns are numbers (do not get out a calculator and start pushing keys for the numbers).

$f^{(0)}(x) =$	$f^{(0)}(x_0) =$	$c_0 =$
$f^{(1)}(x) =$	$f^{(1)}(x_0) =$	$c_1 =$
$f^{(2)}(x) =$	$f^{(2)}(x_0) =$	$c_2 =$
$f^{(3)}(x) =$	$f^{(3)}(x_0) =$	$c_3 =$
$f^{(4)}(x) =$	$f^{(4)}(x_0) =$	$c_4 =$
$f^{(5)}(x) =$	nothing for here	nothing for here

17b Find the  $N^{\text{th}}$ -order Taylor polynomial of  $y = f(x)$  about  $x_0$  in OPEN form for  $N = 0, 1, 2, 3, 4$ .

$P_0(x) =$
$P_1(x) =$
$P_2(x) =$
$P_3(x) =$
$P_4(x) =$

**17c.** Find the Taylor series of  $y = f(x)$  about  $x_0$  in OPEN form.

$$P_{\infty}(x) =$$

**17d.** Find the Taylor series of  $y = f(x)$  about  $x_0$  in CLOSED form.

$$P_{\infty}(x) =$$

**17e.** Consider the given interval  $J$ . Find an upper bound for the maximum of  $|f^{(5)}(x)|$  on the interval  $J$ . Your answer should be a number (leave it as a fraction - do not get out a calculator and start pushing keys). Your answer cannot have an:  $N, x, x_0, c$ .

$$\max_{c \in J} |f^{(5)}(c)| \leq$$

**17f.** Consider the given interval  $J$ . Using Taylor's Remainder Theorem (i.e., Taylor's Big Theorem), find an upper bound for the maximum of  $|R_4(x)|$  on the interval  $J$ . Your answer should be a number (leave it as a fraction - do not get out a calculator and start pushing keys). Your answer cannot have an:  $N, x, x_0, c$ .

$$\max_{x \in J} |R_4(x)| \leq$$

18. Using the fact that

$$\frac{1}{1-r} = \sum_{n=0}^{\infty} r^n \quad \text{when} \quad |r| < 1, \quad (*)$$

find a power series expansion of

$$\frac{x}{4+100x^2}$$

and state when it is valid. Simplify your answer so that your power series has the form

$\sum_{n=0}^{\infty} c_n x^{\text{some power}}$  for some constants  $c_n$ .

$$\frac{x}{4+100x^2} = \sum_{n=0}^{\infty}$$

valid when  $|x| <$