

**Warning: there are 5 problems: 2 problems on the front and 3 problems on the back. Turn your paper over**

NAME: \_\_\_\_\_

There are 5 problems.  
Each problem is worth 2 points.

10

please check the box of your section

Section 005 (WF 8:00 am)

or

Section 006 (WF 9:05 am)

**INSTRUCTIONS:** Indicate your reasoning. Put answers in box and show work below the box.

1.

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} = 0$$

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} = \lim_{n \rightarrow \infty} \left(\frac{1}{2}\right)^n = 0 \leftarrow r = \frac{1}{2}, |r| < 1$$

or

$$\lim_{n \rightarrow \infty} \frac{1}{2^n} = \frac{1}{\infty} = 0$$

2.

$$\lim_{n \rightarrow \infty} (17)^n = \infty \text{ or diverges or DNE}$$

$$\lim_{n \rightarrow \infty} (17)^n = \infty, \quad r = 17, \quad |r| > 1$$

3.

$$\lim_{n \rightarrow \infty} \frac{5n^3 + 6n + 3}{17n^3 + 9n^2 + 4} = \frac{5}{17}$$

$$\lim_{n \rightarrow \infty} \frac{5n^3 + 6n + 3}{17n^3 + 9n^2 + 4} = \lim_{n \rightarrow \infty} \frac{5 + \frac{6}{n^2} + \frac{3}{n^3}}{17 + \frac{9}{n} + \frac{4}{n^3}} = \frac{5 + 0 + 0}{17 + 0 + 0}$$

divide numerator & denom. by  $n^3$  (highest power you see)  $= n^3$

4.

$$\lim_{n \rightarrow \infty} \frac{n^3}{e^n} \stackrel{\frac{\infty}{\infty}}{\text{L'H}} \lim_{n \rightarrow \infty} \frac{3n^2}{e^n} \stackrel{\frac{\infty}{\infty}}{\text{L'H}} \lim_{n \rightarrow \infty} \frac{6n}{e^n} \stackrel{\frac{\infty}{\infty}}{\text{L'H}} \lim_{n \rightarrow \infty} \frac{6}{e^n} = \frac{6}{\infty} = 0$$

did up here

Helpful intuition:  $y = e^x$  grows so much faster than  $y = x^3$  that the answer should be zero. We know, for  $0 < p < \infty$  and  $n$  big enough:  $n^p < e^n$ . So how to show the limit is zero. WAY #1 is to use L'Hôpital's rule. WAY #2 is to use the Squeeze Theorem.

$$0 \leq \frac{n^3}{e^n} \leq \frac{n^3}{n^4} = \frac{1}{n} \xrightarrow{n \rightarrow \infty} 0$$

5.

$$\lim_{n \rightarrow \infty} \frac{\sqrt[3]{n^2 + 5}}{\sqrt[6]{64n^4 + 17n}} = \left(\frac{1}{64}\right)^{1/6} \quad \text{or} \quad \frac{1}{2}$$

hint:

$$\frac{\sqrt[3]{n^2 + 5}}{\sqrt[6]{64n^4 + 17n}} = \frac{(n^2 + 5)^{1/3}}{(64n^4 + 17n)^{1/6}} = \frac{(n^2 + 5)^{2/6}}{(64n^4 + 17n)^{1/6}} = \frac{[(n^2 + 5)^2]^{1/6}}{(64n^4 + 17n)^{1/6}} = \Rightarrow$$

$$\Rightarrow = \left[ \frac{n^4 + 10n^2 + 25}{64n^4 + 17n} \right]^{1/6} = \left[ \frac{1 + \frac{10}{n^2} + \frac{25}{n^4}}{64 + \frac{17}{n^3}} \right]^{1/6} \xrightarrow{n \rightarrow \infty} \left[ \frac{1 + 0 + 0}{64 + 0} \right]^{1/6}$$

÷ num. & denom. by  $n^4$   
(see above problem 3)

NO!

using:  $\lim_{n \rightarrow \infty} (a_n)^p = \left(\lim_{n \rightarrow \infty} a_n\right)^p$

Remark: Unfortunately I saw on too many papers:

$$(n^2 + 5)^{1/3} \stackrel{\text{A}}{=} n^{2/3} + 5^{1/3}. \quad \text{Eq: } \sqrt[n]{a+b} \neq \sqrt[n]{a} + \sqrt[n]{b}.$$