| MARK BOX | | |
|---|---|---|
| PROBLEM | POINTS | |
| 1 a–j | 10 | |
| 2 | 10 | |
| 3a Extra Credit | (1) | |
| 3b | 10 | |
| 3c | 10 | |
| 3d | 10 | |
| 4 | 10 | |
| 5 | 10 | |
| 6 | 10 | |
| 7 | 10 | |
| 8 | 10 | |
| % | 100 | |

**NAME (printed):** Key

**SIGNATURE:** ____________________

**please check the box of your section below**

☐ **Section 005 (W&F 8:00 am)**

or

☐ **Section 006 (W&F 9:05 am)**

**INSTRUCTIONS:**

(1) To receive credit you must:
  (a) **work in a logical fashion, show all your work, indicate your reasoning; no credit will be given for an answer that *just appears*;** such explanations help with partial credit
  (b) when applicable put your answer on/in the line/box provided
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.): Section 7.1 – 7.4, 7.6, 7.7, 8.1 .

**Problem Inspiration:**

**1.** Math 141 HANDOUT, Spring 2006 Exam 1 # 1
**2.** homework problems from § 7.1, , actually problem § 7.1 # 34
**3.** homework problems from § 7.2 and 7.3 , homework problem § Ch. 7 Review # 6, Spring 2006 Exam 1 # 3, and examples from the lectures over § 7.2 & § 7.3
**4.** homework problem § 7.4 # 9 & 11 and Spring 2006 Exam 1 # 4
**5.** homework problem § 8.1 # 6
**6.** homework problem § 8.1 # 6
**7.** homework problem § 8.1 # 11
**8.** homework problem § 8.1 # 15

**1. Fill in the blanks (each worth 1 point).**

If you missed #
1c, 1e, or 1f,
then see the quiz posted on
Ch8 homework page for §8.1

**1a.** $\int \frac{du}{u} = \underline{\ln}\ |u| + C$

**1b.** $\int e^u\,du = \underline{e^u} + C$

**1c.** If $a$ is a constant and $a > 0$ but $a \neq 1$, then

$\int a^u\,du = \underline{\frac{a^u}{\ln a}} + C$

**1d.** $\int \cos u\,du = \underline{\sin u} + C$

**1e.** $\int \tan u\,du = \underline{-\ln|\cos u| + C \text{ or } \ln|\sec u| + C}$

**1f.** $\int \sec u\,du = \underline{\ln|\sec u + \tan u| + C \text{ or } -\ln|\sec u - \tan u|} + C$

**1g.** $\int \sec^2 u\,du = \underline{\tan u} + C$

**1h.** If $a$ is a contant and $a > 0$ then

$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \underline{\sin^{-1}\frac{u}{a}} + C$

**1i.** If $a$ is a contant and $a > 0$ then

$\int \frac{1}{a^2+u^2}\,du = \underline{\frac{1}{a}\tan^{-1}\frac{u}{a}} + C$

**1j.** The integral of $y = f(x)$ with respect to $x$ is denoted by $\int f(x)\,dx$.

The integral of $x = g(y)$ with respect to $y$ is denoted by $\underline{\int g(y)\,dy}$.

Note

$\sin 0 = 0$

$\sin \frac{5\pi}{6} = \frac{1}{2}$

**2.** Let $R$ be the region between by the curve

$$y = \sin x$$

and the line segement jointing the points

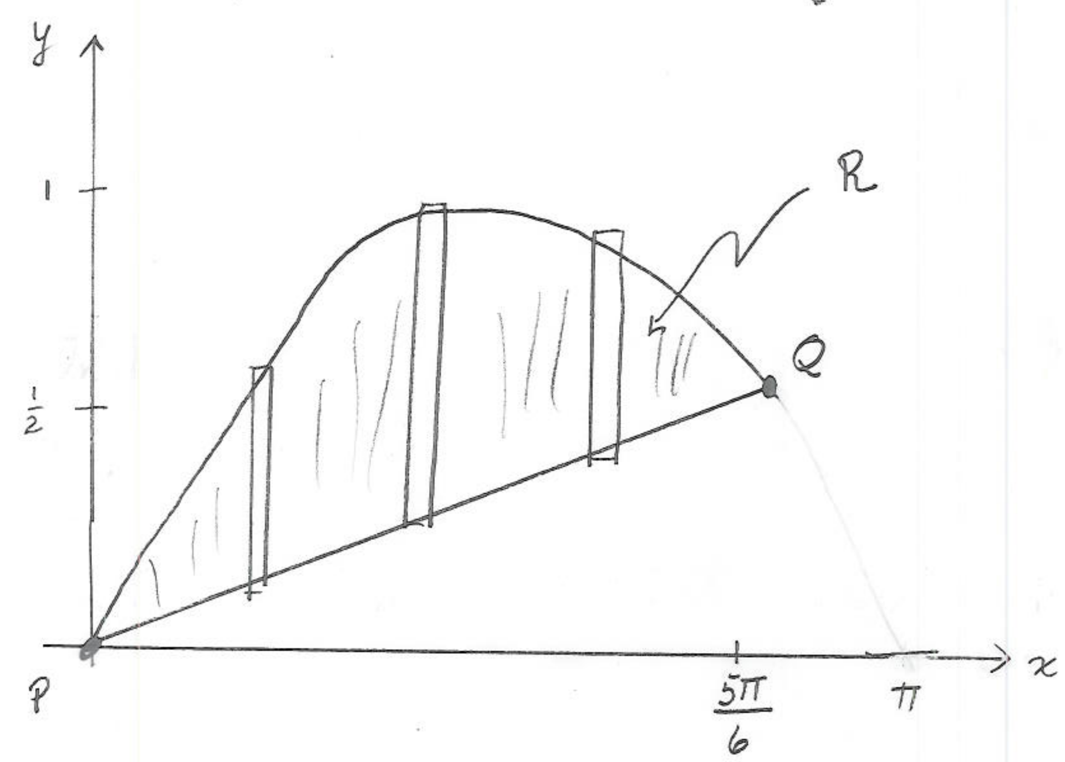
$$P = (\ 0\ ,\ 0\ ) \qquad \text{and} \qquad Q = \left( \frac{5\pi}{6}\ ,\ \frac{1}{2} \right).$$

Let $A$ be the area of the region $R$.

**2a.** Make a rough sketch of the region $R$, labeling $P$ and $Q$.

The equation of the line through points $P$ and $Q$ is: $y = \frac{3x}{5\pi}$.

slope of line thru P & Q $= \frac{\Delta y}{\Delta x} = \frac{\frac{1}{2} - 0}{\frac{5\pi}{6} - 0} = \frac{1}{2} \cdot \frac{6}{5\pi} = \frac{3}{5\pi}$




**2b.** Express the area $A$ as **ONE** integral (but **not** 2 or more integrals). You do NOT have to evaluate the integral(s) nor do lots of algebra.

$$A = \int_{x=0}^{x=\frac{5\pi}{6}} \left( \sin x - \frac{3x}{5\pi} \right) dx$$

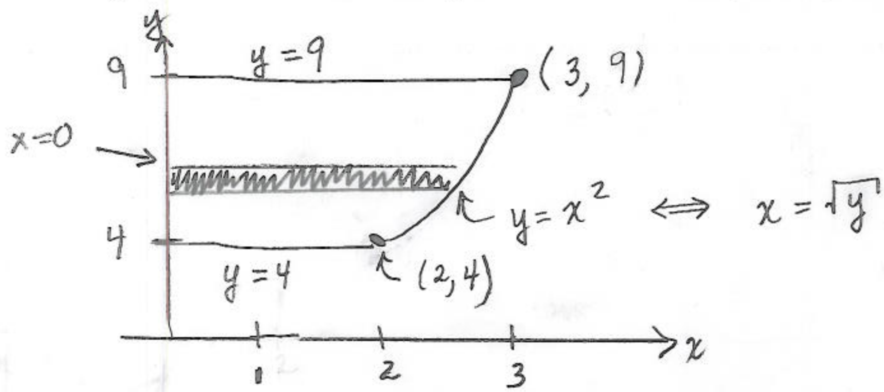
(**3.a**) Sketched below is the region $R$ that is enclosed by

$$x = 0 \quad \text{and} \quad y = 4 \quad \text{and} \quad y = 9 \quad \text{and} \quad y = x^2 .$$

In each of problems ~~**3a**~~, **3b**, **3c**; 3d,

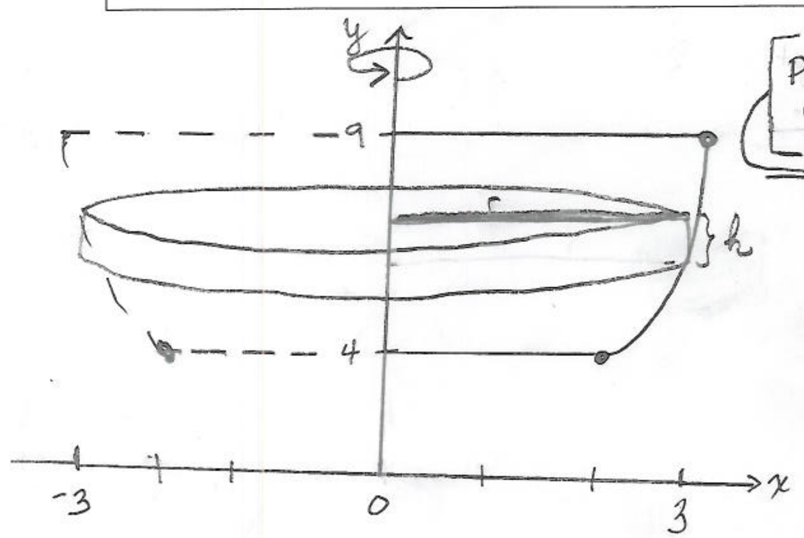
- $R$ will be revolved around some line to create a solid of revolution
- using either the disk, washer, or shell method, express the volume $V$ of the resulting solid of revolution as **one integral** (and NOT as 2 or more integrals).
- In the space provided **below** each problem, make some *good enough sketch* (does not have to be too fancy) to indicate (i.e., help justify) your thinking/reseasoning behind your solution
- you do not have to do lots of algebra to your integrand
- you do not have to integrate your integral.

**Extra Credit/Hint** In the sketch below, draw in a typical rectangle (should it be horizontal or vertical?) that would be used to express the area of $R$ as precisely 1 integral (and not 2 integrals).




3b ~~3a~~. The volume $V$ of the solid obtained by revolving the region $R$ about the $y$-axis is

$$\boxed{V = \int_{y=4}^{y=9} \pi(\sqrt{y})^2\, dy \ \overset{\text{or}}{=}\ \pi \int_{y=4}^{y=9} y\, dy}$$




partition y-axis (to get 1 integral)
revolving about y-axis
no hole
⟹ Disk Method / all in terms of y

Volume of typical element

= volume of a disk

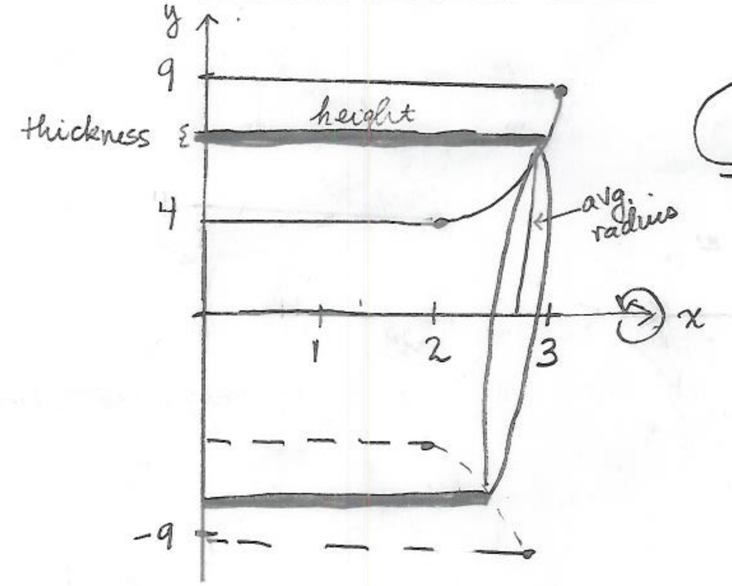
= (area base) (height)

= $(\pi r^2)\quad h$

= $\pi(\sqrt{y})^2 \quad \Delta y$

3c. The volume $V$ of the solid obtained by revolving the region $R$ about the $x$-axis is

$$V = \int_{y=4}^{y=9} 2\pi \, y \cdot \sqrt{y} \, dy \overset{\text{or}}{=} 2\pi \int_{y=4}^{y=9} y^{3/2} \, dy$$




partition y-axis (to get 1 integral)
& revolving about $x$-axis

⇒ Shell Method / all in terms of $y$

Volume of typical element

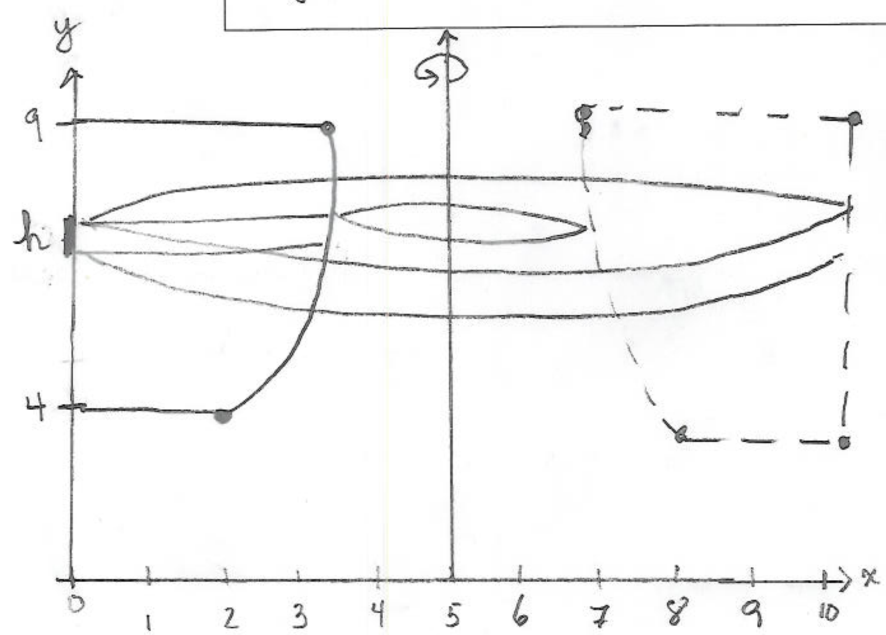
$= $ volume of a shell

$= 2\pi$ (avg. radius)(height)(thickness)

$= 2\pi \, y \sqrt{y} \, \Delta y$

3d. The volume $V$ of the solid obtained by revolving the region $R$ about the vertical line $x = 5$ is

$$V = \int_{y=4}^{y=9} \pi \left(5^2 - (5-\sqrt{y})^2\right) dy \overset{\text{or}}{=} \pi \int_{y=4}^{y=9} \left(10y^{1/2} - y\right) dy$$




partition y-axis (to get 1 integral)
revolve around a line parallel to y-axis
have a hole

⇒ Washer method / all in terms of $y$

Volume of typical element

$= $ volume of a washer

$= \text{Volume}_{big} - \text{Volume}_{little}$

$= \pi r_{big}^2 h - \pi r_{little}^2 h$

$= \pi (r_{big}^2 - r_{little}^2) h$

$= \pi \left(5^2 - (5 - \sqrt{y})^2\right) \Delta y$

**4.** Express the arclength of the parameterized curve

$$x(t) = t^2 + 4$$
$$y(t) = t + 5$$

from the point

$$P = (\ 4\ ,\ 5\ )$$

to the point

$$Q = (\ 13\ ,\ 8\ )$$

as an integral with respect to $t$ .

$$\boxed{\text{arclength} = \int_{t=0}^{t=3} \sqrt{(2t)^2 + 1^2}\ dt \ \overset{\text{or}}{=} \int_{t=0}^{t=3} \sqrt{4t^2+1}\ dt}$$

Once again, you do not have to do lots of algebra to your integrand nor integrate your integral.

$$P = (4,5) \iff \begin{bmatrix} 4 = t^2 + 4 \\ 5 = t + 5 \end{bmatrix} \iff t = 0$$

$$Q = (13,8) \iff \begin{bmatrix} 13 = t^2 + 4 \\ 8 = t + 5 \end{bmatrix} \iff t = 3$$

$$\frac{dx}{dt} = \frac{d}{dt}\left(t^2+4\right) = 2t$$

$$\frac{dy}{dt} = \frac{d}{dt}\left(t+5\right) = 1$$

$$\text{arclength} = \int_{t=a}^{t=b} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\ dt$$

**5.**

$$\boxed{\int \frac{1}{9+25x^2}\,dx = \frac{1}{15}\tan^{-1}\left(\frac{5x}{3}\right)+C \quad \text{or} \quad \frac{1}{15}\arctan\left(\frac{5x}{3}\right)+C}$$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

WAY #1

$$\int \frac{dx}{9+25x^2} = \int \frac{dx}{3^2+(5x)^2} = \frac{1}{5}\int \frac{5\,dx}{3^2+(5x)^2} = \frac{1}{5}\int \frac{du}{3^2+u^2}$$

$$\boxed{\begin{array}{l} u = 5x \\ du = 5\,dx \end{array}}$$

$$= \frac{1}{5}\cdot\frac{1}{3}\tan^{-1}\frac{u}{3} + C = \frac{1}{15}\tan^{-1}\frac{5x}{3} + C$$

$a=3$
See problem 1i

WAY #2

$$\int \frac{dx}{9+25x^2} = \frac{1}{9}\int \frac{dx}{1+\frac{25x^2}{9}} = \frac{1}{9}\int \frac{dx}{1+\left(\frac{5x}{3}\right)^2}$$

want 1 here so

$$= \frac{1}{9}\,\frac{3}{5}\int \frac{\frac{5}{3}\,dx}{1+\left(\frac{5x}{3}\right)^2}$$

$$\boxed{\begin{array}{l} u = \frac{5}{3}x \\ du = \frac{5}{3}\,dx \end{array}}$$

$$= \frac{1}{3\cdot 5}\int \frac{du}{1+u^2} = \frac{1}{15}\tan^{-1} u + C$$

$$= \frac{1}{15}\tan^{-1}\frac{5x}{3} + C$$

Check ans.

$$D_x \tan^{-1} u = \frac{1}{1+u^2}\,\frac{du}{dx}$$

$$D_x\left(\frac{1}{15}\tan^{-1}\frac{5x}{3}\right) = \frac{1}{15}\,\frac{1}{1+\left(\frac{5x}{3}\right)^2}\cdot\frac{5}{3} = \frac{1}{15}\cdot\frac{5}{3}\,\frac{1}{1+\frac{25x^2}{9}}$$

$$= \frac{1}{3\cdot 3}\,\frac{1}{\frac{9+25x^2}{9}} = \frac{1}{3\cdot 3}\,\frac{9}{9+25x^2} = \frac{1}{9+25x^2}$$

**6.** $\boxed{\int \frac{x}{9+25x^2}\,dx = \frac{1}{50} \ln|9+25x^2| + C \overset{\text{or}}{=} \frac{1}{50} \ln(9+25x^2) + C}$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$\boxed{\begin{array}{l} u = 9+25x^2 \\ du = 50x\,dx \end{array}}$ • $9+25x^2$ is always positive so $|9+25x^2| = 9+25x^2$

$$\int \frac{x}{9+25x^2}\,dx = \frac{1}{50}\int \frac{50x\,dx}{9+25x^2} = \frac{1}{50}\int \frac{du}{u} = \frac{1}{50}\ln|u| + C = \frac{1}{50}\ln|9+25x^2| + C$$

$\boxed{\text{Check Answer}}$ $D_x \frac{1}{50} \ln|9+25x^2| = \frac{1}{50} \cdot \frac{1}{9+25x^2} \cdot 50x = \frac{x}{9+25x^2}$ ☺

7. $\boxed{\int \cos^{17}(7x)\ \sin(7x)\ dx = \frac{-1}{7\cdot 18} \cos^{18}(7x) + C \overset{\text{or}}{=} \frac{-1}{126} \cos^{18}(7x) + C}$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$$\boxed{\begin{aligned} u &= \cos(7x) \\ du &= -7\sin(7x)\,dx \end{aligned}}$$

$$\int \cos^{17}(7x)\sin(7x)\,dx = -\frac{1}{7}\int [\cos(7x)]^{17}\left(-7\sin(7x)\,dx\right)$$

$$= -\frac{1}{7}\int u^{17}\,du = -\frac{1}{7}\,\frac{u^{18}}{18} + C = -\frac{1}{7\cdot 18}\left[\cos(7x)\right]^{18}$$

$\boxed{\text{Check Answer}}$ $$D_x\, -\frac{1}{7\cdot 18}\cos^{18}(7x) = -\frac{1}{7\cdot 18}\,18\cos^{17}(7x)\,D_x\cos(7x)$$

$$= -\frac{1}{7\cdot 18}\,18\cos^{17}(7x)\left(-\sin(7x)\right)(7) = \cos^{17}(7x)\sin(7x)$$ ☺

8.

$$\boxed{\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}}\,dx = \frac{2}{3} e^{\sqrt{3x-1}} + C}$$

Remark: box your substitution box for more partial credit.

Recall: you can check your answer via differentiation (if you have time).

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}}\,dx = \int (3x-1)^{-1/2} \, e^{(3x-1)^{1/2}} \, dx$$

WAY #1

$$\boxed{\begin{aligned} u &= (3x-1)^{1/2} \\ du &= \frac{3}{2}(3x-1)^{-1/2}\,dx \end{aligned}}$$

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}}\,dx = \frac{2}{3}\int e^{(3x-1)^{1/2}} \left(\frac{3}{2}(3x-1)^{-1/2}\,dx\right) = \frac{2}{3}\int e^u\,du$$

$$= \frac{2}{3} e^u + C = \frac{2}{3} e^{\sqrt{3x-1}} + C$$

WAY #2

$$\boxed{\begin{aligned} u &= e^{(3x-1)^{1/2}} \\ du &= e^{(3x-1)^{1/2}} \cdot \frac{3}{2}(3x-1)^{-1/2}\,dx \end{aligned}}$$

$$\int \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}}\,dx = \frac{2}{3}\int e^{(3x-1)^{1/2}} \left(\frac{3}{2}(3x-1)^{-1/2}\,dx\right) = \frac{2}{3}\int du$$

$$= \frac{2}{3}u + C = \frac{2}{3} e^{(3x-1)^{1/2}} + C$$

Check Answer

$$D_x \frac{2}{3} e^{\sqrt{3x-1}} = \frac{2}{3} D_x e^{(3x-1)^{1/2}}$$

$$= \frac{2}{3} e^{(3x-1)^{1/2}} \cdot D_x (3x-1)^{1/2} = \frac{2}{3} e^{(3x-1)^{1/2}} \frac{1}{2}(3x-1)^{-1/2}(3)$$

$$= e^{(3x-1)^{1/2}} (3x-1)^{-1/2} = \frac{e^{\sqrt{3x-1}}}{\sqrt{3x-1}}$$