

MARK BOX		
PROBLEM	POINTS	
1 a – r	35	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
13	5	
14	5	
%	100	

NAME: _____

SSN: _____

please check the box of your section below

Section 003 (MW 9:05 pm)

or

Section 004 (MW 10:10 pm)

INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning;**
no credit will be given for an answer that *just appears*;
such explanations help with partial credit
 - (b) when applicable put your answer on/in the line/box provided
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Sections 6.6, 6.8, 6.9, 7.1–7.4, 7.6, 7.7, 8.1–8.8, 10.1–10.10. .

Problem Inspiration: homework and old exams.

Solutions will be available on the course homepage later this afternoon.

1. Fill in the blanks (each worth 1 point) and boxes (each worth 2 points).

1a. $\int \frac{dx}{x} = \underline{\hspace{2cm}} |x| + C$

1b. $D_x e^x = \underline{\hspace{4cm}}$

1c. If $a > 0$ but $a \neq 1$, then $D_x a^x = \underline{\hspace{4cm}}$

Hint: $a^x = e^{\ln(a^x)} = e^{x \ln(a)}$. Your answer should **not** have an “e” in it.

1d. $D_x \tan x = \underline{\hspace{4cm}}$

1e. $\int \sec x \tan x = \underline{\hspace{4cm}} + C$

1f. Integration by parts formula:

$\int u dv = \underline{\hspace{6cm}}$

1g. Trig substitution: (recall that the *integrand* is the function you are integrating)

if the integrand involves $a^2 - u^2$, then one makes the substitution $u = \underline{\hspace{4cm}}$

1h. Trig substitution:

if the integrand involves $a^2 + u^2$, then one makes the substitution $u = \underline{\hspace{4cm}}$

1i. Partial Fraction Decomposition. If one wants to integrate $\frac{f(x)}{g(x)}$ where f and g are polynomials

and $[\text{degree of } f] \geq [\text{degree of } g]$, then one must first do $\underline{\hspace{4cm}}$

1j. **Integral Test:** $a_n > 0$

Let $f: [1, \infty) \rightarrow \mathbb{R}$ be so that

- $a_n = f(\underline{\hspace{2cm}})$ for each $n \in \mathbb{N}$
- f is a $\underline{\hspace{4cm}}$ function
- f is a $\underline{\hspace{4cm}}$ function
- f is a $\underline{\hspace{4cm}}$ function .

Then $\sum a_n$ converges if and only if $\underline{\hspace{4cm}}$ converges.

1k. **Comparison Test:** $a_n > 0$

- If $0 \leq a_n \leq b_n$ for all $n \in \mathbb{N}$ and $\sum b_n \underline{\hspace{2cm}}$, then $\sum a_n \underline{\hspace{2cm}}$.
- If $0 \leq b_n \leq a_n$ for all $n \in \mathbb{N}$ and $\sum b_n \underline{\hspace{2cm}}$, then $\sum a_n \underline{\hspace{2cm}}$.

1l. **Limit Comparison Test:** $a_n > 0$

Let $b_n > 0$ and $\lim_{n \rightarrow \infty} \frac{a_n}{b_n} = L$.

If $\underline{\hspace{2cm}} < L < \underline{\hspace{2cm}}$, then $\sum a_n$ converges if and only if $\sum b_n \underline{\hspace{2cm}}$.

1m. Ratio Test: $a_n > 0$

Let $\rho = \lim_{n \rightarrow \infty} \frac{a_{n+1}}{a_n}$.

- If $\rho < \underline{\hspace{2cm}}$ then $\sum a_n$ converges.
- If $\rho > \underline{\hspace{2cm}}$ then $\sum a_n$ diverges.
- If $\rho = \underline{\hspace{2cm}}$ then the test is inconclusive.

1n. Root Test: $a_n > 0$

Let $\rho = \lim_{n \rightarrow \infty} (a_n)^{\frac{1}{n}}$.

- If $\rho < \underline{\hspace{2cm}}$ then $\sum a_n$ converges.
- If $\rho > \underline{\hspace{2cm}}$ then $\sum a_n$ diverges.
- If $\rho = \underline{\hspace{2cm}}$ then the test is inconclusive.

1o. Alternating Series Test: $a_n > 0$

If

- $a_n \underline{\hspace{2cm}} a_{n+1}$ for each $n \in \mathbb{N}$
- $\lim_{n \rightarrow \infty} a_n = \underline{\hspace{2cm}}$

then $\sum (-1)^n a_n \underline{\hspace{4cm}}$

1p. n^{th} -term test: a_n 's are arbitrary

If $\lim_{n \rightarrow \infty} a_n \neq 0$ or $\lim_{n \rightarrow \infty} a_n$ does not exist, then $\sum a_n \underline{\hspace{4cm}}$.

1q. Consider the interval $I = (a - R, a + R)$ center about $x = a$ and of radius R .

Let $y = f(x)$ be a function that can be differentiated N times $x = a$. Then the N^{th} -order Taylor polynomial $y = P_N(x)$ of f about a is (your answer should have a summation sign \sum in it)

$P_N(x) =$

1r. Consider the interval $I = (a - R, a + R)$ center about $x = a$ and of radius R .

Let $y = f(x)$ be a function that can be differentiated $N + 1$ times for each $x \in I$.

Consider the the N^{th} -order Taylor Remainder term $R_N(x)$, where $f(x) = P_N(x) + R_N(x)$.

Then an upper bound for $|R_N(x)|$ for an $x \in I$ is:

$ R_N(x) \leq$

2.

$$D_x (\cos(\ln x)) =$$

3.

$$D_x 7^{(x^2)} =$$

4.

$$\int (\tan x) (\sec^7 x) dx =$$

Remark: box your substitution box for more partial credit.

5.

$$\int x^2 \arctan x \, dx =$$

Remark: box your substitution box for more partial credit.

6.

$$\int \frac{x^2}{\sqrt{4-x^2}} dx =$$

Remark: box your substitution box for more partial credit.

7. Let R be the region enclosed by

$$y = x^2 \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0 .$$

Let V be the volume of the solid obtained by revolving the region R about the line $x = 3$.

7a. Make a rough sketch below of the region R , labeling the important points.

7b. Using the disk/washer method, express the volume V as an integral (or maybe 2 integrals).
You do NOT have to evaluate the integral(s).

$V =$

8.

$$\lim_{n \rightarrow \infty} \frac{3n^{5/2} + 7n^2 + 9}{-17n^{5/2} + 3n^2 - 9n - 18} =$$

9.

$$\lim_{n \rightarrow \infty} \frac{8n^{15} - 7n^{10} + 19}{-5n^{13} + 6n^8 - 6n^5 + 9} =$$

On problems 10 and 11b, check the correct box and then indicate your reasoning below.
A correctly checked box without appropriate explanation will receive no points.

10. $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n}$

absolutely convergent

conditionally convergent

divergent

11a Let $a_n = \frac{n^3 (n!)}{(2n)!}$. Find $\frac{a_{n+1}}{a_n}$. Simplify your answer so that no factorial sign (i.e., !) appears.

answer: $\frac{a_{n+1}}{a_n} =$

11b. $\sum_{n=17}^{\infty} (-1)^n \frac{n^3 (n!)}{(2n)!}$

absolutely convergent

conditionally convergent

divergent

12. Consider the formal power series

$$\sum_{n=1}^{\infty} \frac{(2x+8)^n}{n}.$$

As we did in class, in the box below draw a diagram indicating for which x 's this series is: absolutely convergent, conditionally convergent, and divergent. Of course, indicate your reasoning.



13. Let

$$f(x) = (1 + x)^{3/2}$$

and $a = 0$.

Find the 3rd-order Taylor polynomial of $y = f(x)$ about (or at) $a = 0$.

$P_3(x) =$

14. As in problem 13, let

$$f(x) = (1 + x)^{3/2}$$

and $a = 0$.

Let $f(x) = P_3(x) + R_3(x)$, where $y = P_3(x)$ is the 3rd-order Taylor polynomial of $y = f(x)$ about $a = 0$ and $y = R_3(x)$ is the corresponding remainder (i.e., error) term.

Consider the interval $I = (-0.5, 0.5)$ center about $a = 0$. Fix an $x \in I$. Find a good upper bound for $|R_3(x)|$.

$ R_3(x) \leq$

Remark: you only have to carry out the algebra as far as I indicated in class.