

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
%	100	

NAME: _____

SSN: _____

please check the box of your section below

 Section 003 (MW 9:05 pm)

or

 Section 004 (MW 10:10 pm)
INSTRUCTIONS:

- (1) To receive credit you must:
 - (a) **work in a logical fashion, show all your work, indicate your reasoning**
 - (b) when applicable put your answer on/in the line/box provided
 - (c) if no such line/box is provided, then box your answer
- (2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.
- (3) You may **not** use a calculator, books, personal notes.
- (4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- (5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Sections 8.1 – 8.5, 8.7, 8.8 .

Problem Inspiration:

1. precisely INTEGRAL HANDOUT homework # 35
2. example from class; precisely INTEGRAL HANDOUT homework # 23
3. example from class and homework problem § 8.3 # 3; , precisely problem § 8.3 # 4
4. precisely INTEGRAL HANDOUT homework # 1
5. example from class, homework problem § 8.2 # 1,7; , precisely problem § 8.2 # 4
6. precisely homework problem § 8.4 # 35
7. homework problem § 8.5 # 17; , precisely problem § 8.5 # 18
8. homework problem § 8.7 # 1
9. homework problem § 8.7 # 7
10. homework from 8.8

1.

$$\int \sec^3 x \tan^3 x dx = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$$u = \sec x$$

$$du = \sec x \tan x dx$$

$$\frac{\cos^2 x}{\cos^2 x} + \frac{\sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$$

$$1 + \tan^2 x = \sec^2 x$$

$$\int \sec^3 x \tan^3 x dx = \int \sec^2 x \tan^2 x (\sec x \tan x dx)$$

$$= \int \sec^2 x (\sec^2 x - 1) (\sec x \tan x dx)$$

$$= \int u^2 (u^2 - 1) du$$

$$= \int (u^4 - u^2) du$$

$$= \frac{u^5}{5} - \frac{u^3}{3} + C$$

2.

$$\int \ln(1+x) dx = (1+x) \ln(1+x) - x + C \quad \text{or} \quad x \ln(1+x) - x + \ln(1+x) + C$$

WAY #1

$$\begin{array}{ll} u = \ln(1+x) & dv = dx \\ du = \frac{dx}{1+x} & v = x \end{array}$$

$$\int \ln(1+x) dx = x \ln(1+x) - \int \frac{x}{1+x} dx$$

$$\stackrel{\text{L.D.}}{=} x \ln(1+x) - \int \left(1 - \frac{1}{1+x}\right) dx$$

$$= x \ln(1+x) - x + \ln(1+x) + C$$

$$= \underbrace{(1+x)}_{\downarrow} \ln(1+x) - x + C$$

WAY #2 - easier!

$$u = \ln(1+x) \quad dv = dx$$

$$du = \frac{dx}{1+x} \quad v = 1+x$$

$$\int \ln(1+x) dx = (1+x) \ln(1+x) - \int (1+x) \frac{dx}{(1+x)}$$

$$= (1+x) \ln(1+x) - \int dx$$

$$= (1+x) \ln(1+x) - x + C$$

3.

$$\int \cos^2(3x) dx = \frac{x}{2} + \frac{1}{12} \sin(6x) + C$$

$$\int \cos^2(3x) = \int \frac{1 + \cos 6x}{2} dx$$

$$= \frac{1}{2} \left[\int dx + \frac{1}{6} \int \cos 6x (6 dx) \right]$$

$$= \frac{1}{2} \left[x + \frac{1}{6} \sin 6x \right] + C$$

4.

$$\int \frac{dx}{\sqrt{x}(1+x)} = 2 \tan^{-1} \sqrt{x} + C$$

Hint: $1+x = 1+(\sqrt{x})^2$

$$\begin{array}{l} u = x^{1/2} \\ u^2 = x \end{array} \quad \begin{array}{l} \longrightarrow \\ \text{or} \\ \longrightarrow \end{array} \quad \begin{array}{l} du = \frac{1}{2} x^{-1/2} dx \\ 2u du = dx \end{array}$$

$$\int \frac{dx}{\sqrt{x}(1+x)} = \int \frac{1}{1+(\sqrt{x})^2} (x^{-1/2} dx)$$

$$= \int \frac{1}{1+u^2} (2 du) = 2 \int \frac{du}{1+u^2}$$

$$= 2 \tan^{-1} u + C$$

$$5. \int x^2 e^{-2x} dx = \frac{-x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

$$u_1 = x^2 \quad dv_1 = e^{-2x} dx$$

$$du_1 = 2x dx \quad v_1 = -\frac{1}{2} e^{-2x}$$

$$u_2 = x \quad dv_2 = e^{-2x} dx$$

$$du_2 = dx \quad v_2 = -\frac{1}{2} e^{-2x}$$

$$\int x^2 e^{-2x} dx \stackrel{\textcircled{1}}{=} -\frac{x^2}{2} e^{-2x} - \int -\frac{1}{2} e^{-2x} 2x dx$$

$$= -\frac{x^2}{2} e^{-2x} + \int x e^{-2x} dx$$

$$\stackrel{\textcircled{2}}{=} -\frac{x^2}{2} e^{-2x} + \left[-\frac{x}{2} e^{-2x} - \int -\frac{1}{2} e^{-2x} dx \right]$$

$$= -\frac{x^2}{2} e^{-2x} - \frac{x}{2} e^{-2x} + \frac{1}{2} \left(-\frac{1}{2}\right) \int e^{-2x} (-2 dx)$$

$$= -\frac{x^2 e^{-2x}}{2} - \frac{x e^{-2x}}{2} - \frac{e^{-2x}}{4} + C$$

6.

$$\int \frac{dx}{\sqrt{3+2x-x^2}} = \sin^{-1} \left(\frac{x-1}{2} \right) + C$$

Hint: complete the square.

$$3 + 2x - x^2 = - [x^2 - 2x - 3]$$

$$= - \left[\underbrace{(x-1)^2}_{\downarrow} - 4 \right]$$

$$\underbrace{x^2 - 2x + 1}_{\downarrow} \quad \underbrace{-4}_{\uparrow}$$

$$= 4 - (x-1)^2$$

$$\int \frac{dx}{\sqrt{3+2x-x^2}} = \int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{du}{\sqrt{2^2-u^2}}$$

$$u = x - 1$$

$$du = dx$$

$$a = 2$$

$$= \sin^{-1} \frac{u}{2} + C$$

7.

$$\int \frac{x^2}{x^2 - 3x + 2} dx = x - \ln|x-1| + 4 \ln|x-2| + C$$

Hint: bigger bottoms? \rightarrow NO so do long division

$$x^2 - 3x + 2 \overline{) \begin{array}{r} x^2 + 0x + 0 \\ - (x^2 - 3x + 2) \\ \hline 3x - 2 \end{array}} \Rightarrow \frac{x^2}{x^2 - 3x + 2} = 1 + \frac{3x - 2}{x^2 - 3x + 2}$$

$$\frac{3x - 2}{x^2 - 3x + 2} = \frac{3x - 2}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{A(x-2) + B(x-1)}{(x-1)(x-2)}$$

$$\Rightarrow 3x - 2 = A(x-2) + B(x-1)$$

equate
coeff : x $3 = A + B$
constant $-2 = -2A - B$

$$\oplus \quad 1 = -A \Rightarrow \boxed{A = -1} \Rightarrow \boxed{B = 4}$$

$$\Rightarrow \int \frac{x^2}{x^2 - 3x + 2} dx = \int \left[1 + \frac{-1}{x-1} + \frac{4}{x-2} \right] dx$$

8. Use the trapezoidal rule, with $n = 6$ subintervals, to approximate the integral

$$\int_2^5 \sqrt{x-1} \, dx .$$

ANSWER: T_6 is equal to:

$$\frac{1}{4} \left[1 + 2\sqrt{1.5} + 2\sqrt{2} + 2\sqrt{2.5} + 2\sqrt{3} + 2\sqrt{3.5} + \sqrt{4} \right]$$

You only have to do arithmetic as far as I indicated in class.

$$T_n = \frac{b-a}{2n} \left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n) \right]$$

$$\Delta x = \frac{b-a}{n} = \frac{5-2}{6} = \frac{3}{6} = \frac{1}{2} = .5$$

x_0	x_1	x_2	x_3	x_4	x_5	x_6
2	$2.5 = \frac{5}{2}$	$3 = \frac{6}{2}$	$3.5 = \frac{7}{2}$	$4 = \frac{8}{2}$	$4.5 = \frac{9}{2}$	5

$$T_6 = \underbrace{\frac{5-2}{2(6)}}_{\frac{1}{4}} \left[\sqrt{2-1} + 2\sqrt{2.5-1} + 2\sqrt{3-1} + 2\sqrt{3.5-1} + 2\sqrt{4-1} + 2\sqrt{4.5-1} + \sqrt{5-1} \right]$$

9. Recall the Trapezoidal Error Bound Theorem: If f'' is continuous on $[a, b]$ and if K_2 is the maximum value of $|f''(x)|$ on $[a, b]$, then

$$\left| T_n - \int_a^b f(x) dx \right| \leq \frac{(b-a)^3 K_2}{12n^2}. \quad (11)$$

Use formula (11) to find an upper bound on the error in problem 8.

Answers:

$$K_2 = \frac{1}{4}$$

$$\left| T_6 - \int_2^5 \sqrt{x-1} dx \right| \leq \frac{1}{64}$$

You only have to do arithmetic as far as I indicated in class.

$$\left. \begin{aligned} f(x) &= (x-1)^{1/2} \\ f'(x) &= \frac{1}{2}(x-1)^{-1/2} \\ f''(x) &= -\frac{1}{4}(x-1)^{-3/2} \end{aligned} \right\} \Rightarrow K_2 = \max_{2 \leq x \leq 5} \left| -\frac{1}{4}(x-1)^{-3/2} \right|$$

$$= \frac{1}{4} \max_{2 \leq x \leq 5} \frac{1}{(x-1)^{3/2}}$$

$$\stackrel{x=2}{=} \frac{1}{4} \frac{1}{(2-1)^{3/2}} = \frac{1}{4} \cdot \frac{1}{1^{3/2}} = \frac{1}{4}$$

$$\left| T_6 - \int_2^5 \sqrt{x-1} dx \right| \leq \frac{(5-2)^3}{12(6)^2} \cdot \frac{1}{4}$$

$$= \frac{3^3}{12(6)^2 \cdot 4} \quad \leftarrow \text{to here is ok}$$

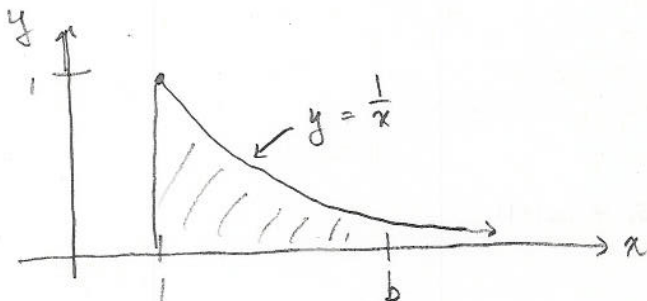
$$= \frac{3^3}{3 \cdot 4 \cdot 3 \cdot 2 \cdot 3 \cdot 2 \cdot 2} \cdot \frac{1}{4}$$

$$= \frac{1}{4} \cdot 4 \cdot \frac{1}{4} = \frac{1}{4^3} = \frac{1}{64}$$

10.

$$\int_1^{\infty} \frac{dx}{x} = \infty \text{ or } [\text{does not exist}] .$$

Work logically and clearly so that I can see that you see what is going on. Explain your answer.



$$\int_1^{\infty} \frac{dx}{x} = \lim_{b \rightarrow \infty} \int_1^b \frac{dx}{x}$$

$$= \lim_{b \rightarrow \infty} \left[\ln x \Big|_{x=1}^{x=b} \right]$$

$$= \lim_{b \rightarrow \infty} \left[\ln b - \underbrace{\ln 1}_0 \right]$$

$$= \lim_{b \rightarrow \infty} \ln b$$

$$= \infty$$