

1. Use the Fundamental Theorem of Calculus to find:

Solutions

$$\frac{d}{dx} \int_{17}^x e^{\sqrt{t+5}} dt = e^{\sqrt{x+5}}$$

2. Evaluate the below derivative.

$$\frac{d}{dx} 7^{(x^2)} = (7^{x^2}) (\ln 7) (2x)$$

3. Evaluate the below integral. (Hint: factor $3x + 6$.)

$$\int_{x=1}^{x=2} \frac{3x+6}{\sqrt{x^2+4x+7}} dx = 3 (\sqrt{19} - \sqrt{12})$$

$$\int_{x=1}^{x=2} \frac{3x+6}{\sqrt{x^2+4x+7}} dx = 3 \int_{x=1}^{x=2} (x^2+4x+7)^{-1/2} (x+2) dx$$

$$= \frac{3}{2} \int_{u=12}^{u=19} u^{-1/2} du$$

$$= \frac{3}{2} \cdot \frac{2}{1} u^{1/2} \Big|_{u=12}^{u=19} = 3 [\sqrt{19} - \sqrt{12}]$$

$$\underline{\underline{or}} \quad 3 [\sqrt{19} - 2\sqrt{3}]$$

4. Evaluate the below integral. Your answer should be a real number and should not involve any log, trig, etc, functions.

$$\int_{x=-\frac{1}{\sqrt{3}}}^{x=0} \frac{dx}{1+9x^2} = \boxed{\frac{\pi}{9}}$$

$$\int_{x=-\frac{1}{\sqrt{3}}}^{x=0} \frac{dx}{1+9x^2} = \int_{x=-\frac{1}{\sqrt{3}}}^{x=0} \frac{dx}{1+(3x)^2} = \frac{1}{3} \int_{u=-\frac{3}{\sqrt{3}}}^{u=0} \frac{du}{1^2+u^2}$$

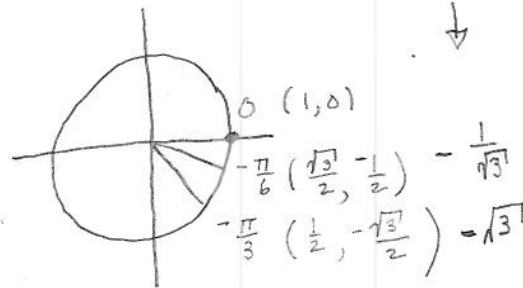
$$u = 3x \\ du = 3 dx$$

$$= \frac{1}{3} \tan^{-1} \Big|_{u=-\frac{3}{\sqrt{3}}}^{u=0} = -\frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = -\sqrt{3}$$

$$= \frac{1}{3} [\tan^{-1} 0 - \tan^{-1} (-\sqrt{3})]$$

$$= \frac{1}{3} [0 - \frac{-\pi}{3}]$$

$$= \frac{\pi}{9}$$



$$\frac{\cot}{a} \\ \frac{1}{n}$$

5. The Logarithmic Function Time.

5a. Fill in the one box. By definition, for $x > 0$,

$$\ln x = \int_1^{\boxed{x}} \frac{dt}{t}.$$

5b. Let $\ln a = 3$ and $\ln c = 6$. Then

$$\int_1^{\frac{a}{c}} \frac{dt}{t} = \boxed{-3}.$$

Your answer should be a number and should not have a \ln , a , or c in it.

$$\int_1^{a/c} \frac{dt}{t} = \ln\left(\frac{a}{c}\right) = \ln a - \ln c = 3 - 6 = -3$$

(5) This exam covers (from *Calculus* by Anton, Bivens, Davis 8th ed.):
Section 6.6, 6.8, 6.9, 7.1 – 7.4, 7.6, 7.7 .

Problem Inspiration:

1. homework problem § 6.6 # 55
 2. homework problem § 6.9 # an additional problem
 3. homework problem § 6.8 # 33
 4. homework problem § 6.8 # 47
 5. homework problem § 6.9 # 3
 6. homework problem § 7.1 # 31 , actually problem § 7.1 # 32
 7. homework problem § 7.2 # 7 , actually problem § 7.2 # 10
 8. homework problem § 7.3 # 5 , actually problem § 7.3 # 6
 9. homework problem § 7.4 # 7 , actually problem § 7.4 # 8
 10. combines ideas from § 7.2 and 7.3
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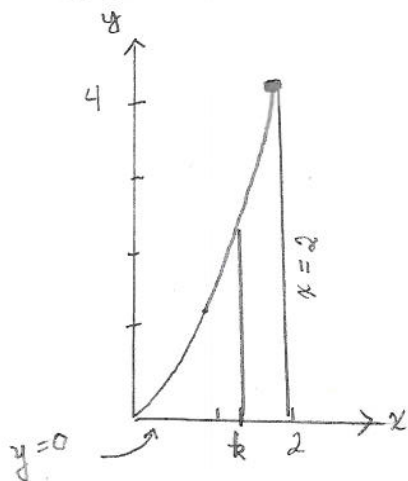
6. Find a vertical line $x = k$ that divides the area enclosed by

$$x = \sqrt{y} \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0$$

into two equal parts.

$$x^2 = y$$

ANSWER: the vertical line is $x = \sqrt[3]{4}$.



WANT $\int_0^k x^2 dx = \int_k^2 x^2 dx$

$$\Leftrightarrow \frac{1}{3} x^3 \Big|_{x=0}^{x=k} = \frac{1}{3} x^3 \Big|_{x=k}^{x=2}$$

$$\Leftrightarrow \frac{1}{3} (k^3 - 0^3) = \frac{1}{3} (2^3 - k^3)$$

$$\Leftrightarrow k^3 = 8 - k^3$$

$$\Leftrightarrow 2k^3 = 8$$

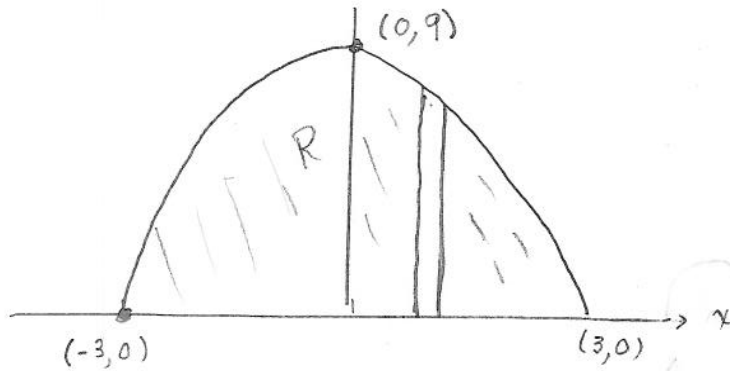
$$\Leftrightarrow k^3 = 4$$

7. Let R be the region enclosed by

$$y = 9 - x^2 \quad \text{and} \quad y = 0.$$

Let V be the volume of the solid obtained by revolving the region R about the x -axis.

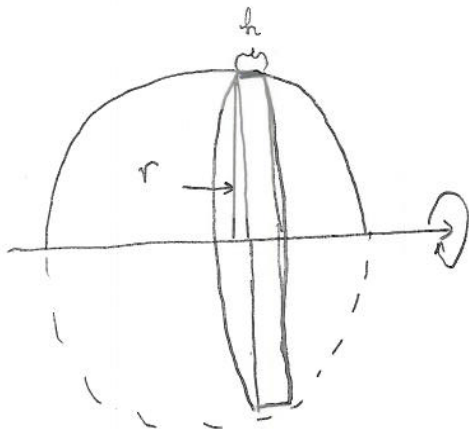
7a. Make a rough sketch of the region R , labeling the important points.



7b. Using the disk/washer method, express the volume V as an integral (or maybe 2 integrals).

You do NOT have to evaluate the integral(s).

$$V = \int_{x=-3}^{x=3} \pi (9-x^2)^2 dx$$



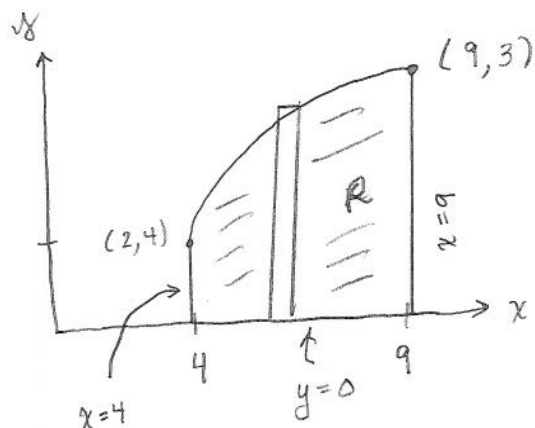
$$\begin{aligned} V_{\text{typical element}} &= V \cdot \text{of "tuna can" or disk} \\ &= \pi r^2 h \\ &= \pi (9-x^2)^2 \Delta x \end{aligned}$$

8. Let R be the region enclosed by

$$y = \sqrt{x} \quad \text{and} \quad x = 4 \quad \text{and} \quad x = 9 \quad \text{and} \quad y = 0.$$

Let V be the volume of the solid obtained by revolving the region R about the y -axis.

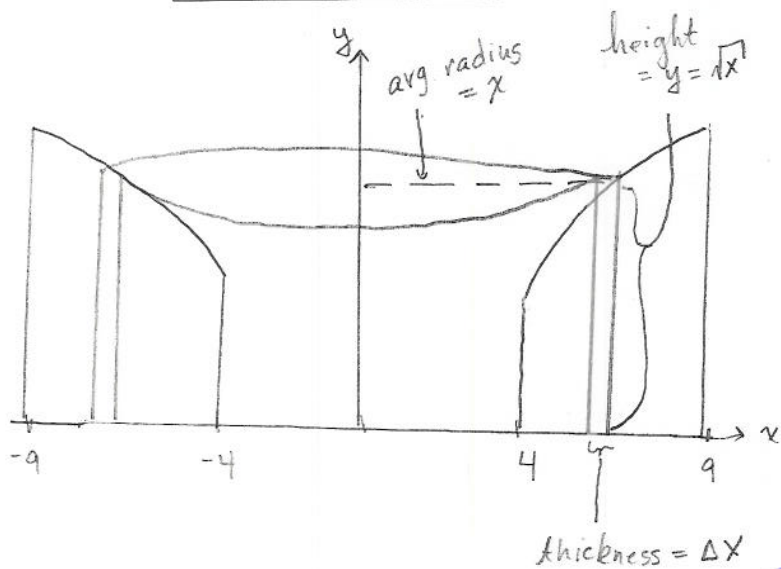
8a. Make a rough sketch of the region R , labeling the important points.



8b. Using the cylindrical shell method, express the volume V as an integral (or maybe 2 integrals).

You do NOT have to evaluate the integral(s).

$$V = \int_{x=4}^{x=9} 2\pi x^{3/2} dx$$



$$\begin{aligned} V & \text{ of typical element} \\ & = V \text{ of shell} \\ & = 2\pi (\text{average radius})(\text{thickness})(\text{height}) \\ & = 2\pi x \Delta x \sqrt{x} \\ & = 2\pi x \sqrt{x} \Delta x \\ & \text{or} \quad 2\pi x^{3/2} \Delta x \end{aligned}$$

(7)

9. Let L be the arc length of the curve

$$x = \frac{y^4}{8} + \frac{y^{-2}}{4} \quad \text{from } y = 1 \quad \text{to} \quad y = 4.$$

Express L as an integral You do NOT have to evaluate the integral.

ANSWER: $L = \int_{y=1}^{y=4} \sqrt{1 + \left(\frac{1}{2}y^3 - \frac{1}{2}y^{-3}\right)^2} dy$

$$g(x) = \frac{1}{8}y^4 + \frac{1}{4}y^{-2}$$

$$g'(x) = \frac{1}{2}y^3 - \frac{1}{2}y^{-3}$$

$$[g'(x)]^2 = \frac{y^6}{4} - \frac{1}{2} + \frac{y^{-6}}{4}$$

$$1 + [g'(x)]^2 = \frac{y^6}{4} + \frac{1}{2} + \frac{y^{-6}}{4} = \frac{1}{4} (y^3 + y^{-3})^2$$

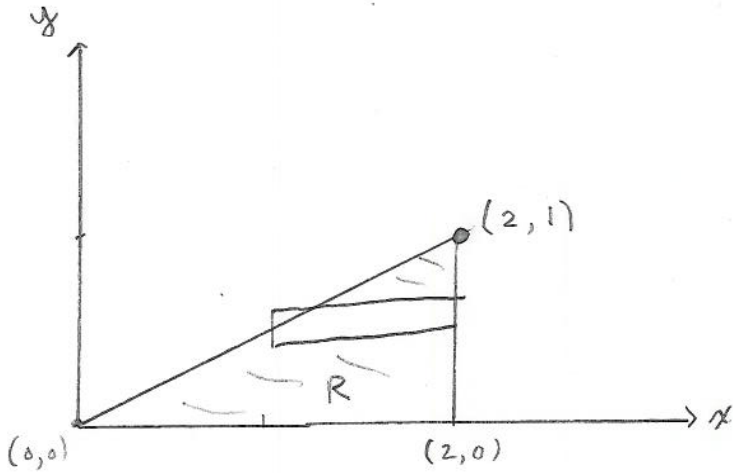
$$\sqrt{1 + [g'(x)]^2} = \frac{1}{2} (y^3 + y^{-3})$$

10. Let R be the region enclosed by

$$y = \frac{x}{2} \quad \text{and} \quad x = 2 \quad \text{and} \quad y = 0.$$

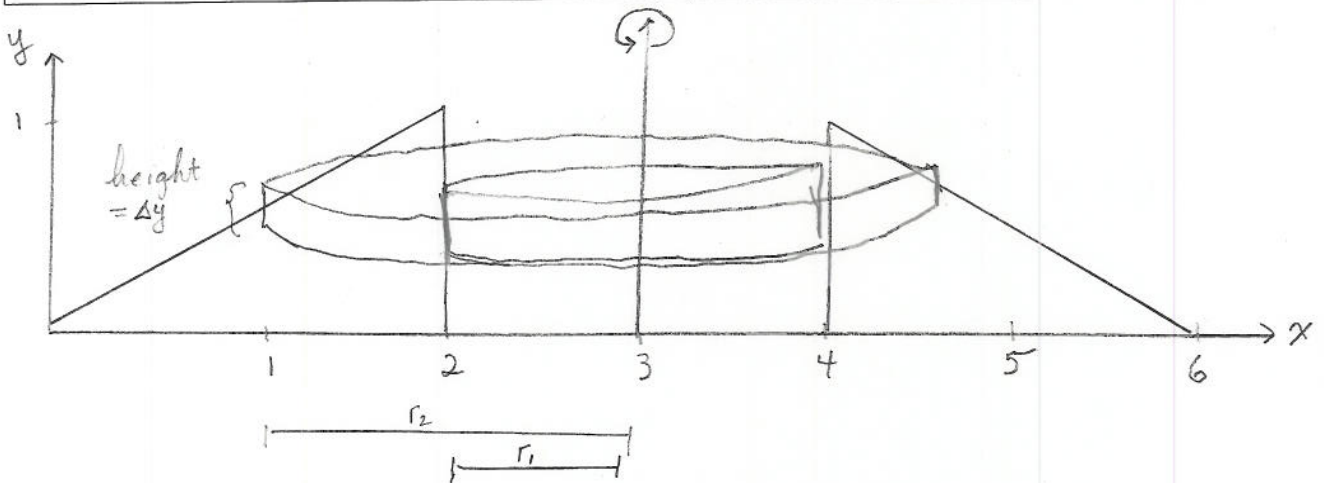
Let V be the volume of the solid obtained by revolving the region R about the line $x = 3$.

10a Make a rough sketch of the region R , labeling the important points.



10b Using the ~~disk~~/washer method, express the volume V as an integral (or maybe 2 integrals). You do NOT have to evaluate the integral(s).

$$V = \int_{y=0}^{y=1} \pi \left[(3-2y)^2 - 1^2 \right] dy \quad \text{or} \quad \pi \int_{y=0}^{y=1} (4y^2 - 12y + 8) dy$$



$$\begin{aligned} \text{Volume of typical element} &= \pi (r_2^2 - r_1^2) (\text{height}) \\ &= \pi ((3-2y)^2 - 1^2) \Delta y \end{aligned}$$