Prof. Girardi	Math 142.001/002	Spring 2003	03.10.03	Exam 3

MARK BOX		
PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
%	100	

NAME: Key

SSN: ____________________

Section 001 (MW 9:05)

or

Section 002 (MW 10:10)

INSTRUCTIONS:

(1) To receive credit you must:
  (a) work in a logical fashion, show all your work, indicate your reasoning
  (b) when applicable put your answer on/in the line/box provided
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes. Give exact answers: for example, write $\ln 2$ instead of .6931, write $\sqrt{2}$ instead of 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.

(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Varberg, Purcell, Rigdon, 8th ed.): Sectionss 10.1-10.5 .

ADDITIONAL INSTRUCTIONS

In problems 1 - 5, find the limit of each sequence.
In problems 7 - 10, check the appropriate box of what that series is doing.
Always justify your answer.

Problem Inspiration:

1-3. An idea we have gone over-and-over in class.
4-5. examples from class
6. just changed numbers from an example from class
7-10. minor variations of problems from SERIOUS SERIES PROBLEMS.

1. $\lim\limits_{n\to\infty} \frac{12n^2-3n+1}{3n^2-17n+100} = \lim\limits_{n\to\infty} \frac{12-\frac{3}{n}+\frac{1}{n^2}}{3-\frac{17}{n}+\frac{100}{n^2}} = \frac{12-0+0}{3-0+0} = \boxed{4}$

or.

$\lim\limits_{n\to\infty} \frac{\boxed{12}\,n^{2}-3n+1}{\boxed{3}\,n^{2}-17n+100} = \frac{12}{3} = 4.$ same

2. $\lim\limits_{n\to\infty} \frac{3n^2+4}{n-1} = \lim\limits_{n\to\infty} \frac{3+\frac{4}{n^2}}{\frac{1}{n}-\frac{1}{n^2}} = \cancel{\frac{3+0}{0-0}} = \cancel{\frac{3}{0}} = \boxed{\infty}$ or DNE or diverges

3. $\lim\limits_{n\to\infty} \frac{n-1}{3n^2+4} = \lim\limits_{n\to\infty} \frac{\frac{1}{n}-\frac{1}{n^2}}{3+\frac{4}{n^2}} = \frac{0-0}{3+0} = \frac{0}{3} = \boxed{0}$

4. $\lim\limits_{n\to\infty} (0.9999)^n = 0$ since $\lim\limits_{n\to\infty} r^n = \begin{cases} 0 & |r|<1 \\ 1 & r=1 \\ \text{DNE} & |r|>1 \text{ or } r=-1 \end{cases}$

we did in class

5. $\lim\limits_{n\to\infty} \frac{n}{e^n} = 0$ since $\lim\limits_{x\to\infty} \frac{x}{e^x} \underset{L'H}{\overset{\frac{\infty}{\infty}}{=}} \lim\limits_{x\to\infty} \frac{1}{e^x} = 0$

or you could say "since exp. grows so much faster than poly..."

6. $\sum\limits_{n=3}^{\infty} 9\left(\frac{1}{10}\right)^n = 9\left[\left(\frac{1}{10}\right)^3 + \left(\frac{1}{10}\right)^4 + \left(\frac{1}{10}\right)^5 + \cdots\right]$

want 1 here so factor out $\left(\frac{1}{10}\right)^3$

$= 9\cdot\frac{1}{10^3}\left[1+\left(\frac{1}{10}\right)^1+\left(\frac{1}{10}\right)^5+\cdots\right] = \frac{9}{10^3}\,\frac{1}{1-\frac{1}{10}} = \frac{9}{10^3}\cdot\frac{10}{9} = \frac{1}{10^2} = \boxed{\frac{1}{100}}$

7. $\sum (-1)^n \frac{n+1}{n!}$ $\boxed{\text{abs. conv.}}$

Consider $\sum \left|(-1)^n \frac{n+1}{n!}\right| = \sum \frac{n+1}{n!}$ factorial ⇒ ratio test

$\lim\limits_{n\to\infty} \frac{(n+1)+1}{(n+1)!}\cdot\frac{n!}{n+1} = \lim\limits_{n\to\infty} \frac{n+2}{n+1}\,\frac{n!}{n!(n+1)} = \lim\limits_{n\to\infty} \frac{n+2}{n+1}\cdot\frac{1}{n+1} = \frac{1}{1}\cdot 0 = 0$

8. $\sum\limits_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$ $\boxed{\text{cond. conv}}$

Consider $\sum \left|(-1)^n \frac{1}{\sqrt{n}}\right| = \sum \frac{1}{n^{1/2}}$ ← div p series, $p=\frac{1}{2}\le 1$

AST $\sum(-1)^n a_n$ where $a_n = \frac{1}{n^{1/2}}$, Clearly, a_n decr. & $\lim\limits_{n\to\infty} a_n = 0 \Rightarrow \sum (-$

9. $\sum (-1)^n \frac{\sin n}{n^2+1}$ $\boxed{\text{abs. conv.}}$

$\left|(-1)^n \frac{\sin n}{n^2+1}\right| = \frac{|\sin n|}{n^2+1} \le \frac{1}{n^2}$. $\sum \frac{1}{n^2}$ conv (p series, $p=2>1$).

Comparison Test $\Rightarrow \sum \left|(-1)^n \frac{\sin n}{n^2+1}\right|$ conv.

10. $\sum (-1)^n \frac{n^2+1}{3n^2+n-1}$ $\boxed{\text{div}}$ by n^{th} term test for divergence.

$\lim (-1)^n \frac{n^2+1}{3n^2+n-1}$ DNE $\neq 0$