Prof. Girardi Math 142.001/002 Spring 2003 03.20.03 Exam 2

MARK BOX

PROBLEM	POINTS	
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
%	100	

NAME: key

SSN: ______________

Section 001 (MW 9:05)

or

Section 002 (MW 10:10)

INSTRUCTIONS:

(1) To receive credit you must:
  (a) work in a logical fashion, show all your work, indicate your reasoning
  (b) when applicable put your answer on/in the line/box provided
  (c) if no such line/box is provided, then box your answer

(2) The MARK BOX indicates the problems along with their points.
Check that your copy of the exam has all of the problems.

(3) You may **not** use a calculator, books, personal notes. Give exact answers: for example, write $\ln 2$ instead of .6931, write $\sqrt{2}$ instead of 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.

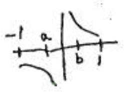
(4) During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.

(5) This exam covers (from *Calculus* by Varberg, Purcell, Rigdon, 8th ed.): Chapters 8 and 9 .

Problem Inspiration:

1. # 1 of class handout of 100 integrals, also an example from class
2. # 10 of class handout of 100 integrals
3. an example from class
4. an example from class
5. # 47 of class handout of 100 integrals, also an example from class
6. # 44 of class handout of 100 integrals
7. an example from class
8. homework problem § 9.2 # 23
9. an example from class
10. homework problem § 9.4 # 7

(10)



$$\int_{-1}^{1} \frac{dx}{x^3} = \lim_{a\to 0^-} \int_{-1}^{a} x^{-3}\,dx + \lim_{b\to 0^+} \int_{b}^{1} x^{-3}\,dx$$

$$= \lim_{a\to 0^-} \frac{x^{-2}}{-2}\Big|_{x=-1}^{x=a} + \lim_{b\to 0^+} \frac{x^{-2}}{-2}\Big|_{x=b}^{x=1}$$

$$= \underbrace{\lim_{a\to 0^-} \left[-\frac{1}{2a^2} + \frac{1}{2}\right]}_{-\infty} + \underbrace{\lim_{b\to 0^+} \left[-\frac{1}{2} + \frac{1}{2b^2}\right]}_{\infty}$$

1.

$u = x^{1/2}$
$u^2 = x$
$2u\,du = dx$

$$\int \frac{dx}{\sqrt{x}\,(1+x)} = \int \frac{2u\,du}{u\,(1+u^2)} = 2\int \frac{du}{1+u^2} = 2\arctan u + C = \boxed{2\arctan\sqrt{x} + C}$$

2.

$$\int \frac{dx}{\sqrt{x^2+4}} = \ln\left|\frac{\sqrt{x^2+4}}{2} + \frac{x}{2}\right| + C \overset{\text{or}}{=} \ln\left|\sqrt{x^2+4} + x\right| \underbrace{- \ln 2 \quad + C}_{\tilde{C}}$$

3.

$$\int e^x \cos x\,dx = \frac{e^x\,(\cos x + \sin x)}{2} \quad +C$$

4.

$u = \ln x$ $dv = dx$
$du = \frac{dx}{x}$ $v = x$

$$\int \ln x\,dx = x\ln x - \int \frac{x\,dx}{x} = x\ln x - \int dx = \boxed{x\ln x - x \quad + C}$$

5.

$$\int \frac{x^4+2x+2}{x^5+x^4}\,dx = \int \left(2x^{-4} + \frac{1}{x+1}\right) dx = \boxed{\frac{2x^{-3}}{-3} + \ln|x+1| \qquad +C}$$

6.

$$\int \frac{4x^3-x+1}{x^3+1}\,dx = 4x - \frac{2}{3}\ln|x+1| + \frac{1}{3}\ln|x^2-x+1| - \frac{4\sqrt{3}}{3}\tan^{-1}\left(\frac{2x}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) + C$$

7.

$$\lim_{x\to\infty} \frac{x^2}{e^x} \underset{\text{L'H}}{\overset{\cancel{\frac{\infty}{\infty}}}{=}} \lim_{x\to\infty} \frac{2x}{e^x} \underset{\text{L'H}}{\overset{\cancel{\frac{\infty}{\infty}}}{=}} \lim_{x\to\infty} \frac{2}{e^x} \underset{\overset{\uparrow}{\cancel{\frac{\infty}{\infty}}}}{=} 0.$$

8.

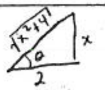
8. $\lim_{x\to\infty} x^{\frac{1}{x}} = 1.$

9. $\int_1^\infty \frac{dx}{x^2} = \lim_{b\to\infty} \int_1^b x^{-2}\,dx = \lim_{b\to\infty} -\frac{1}{x}\Big|_{x=1}^{x=b} = \lim_{b\to\infty} -\frac{1}{b} - \frac{-1}{1} = 1$

10. $\int_{-1}^1 \frac{dx}{x^3} =$ diverges - DNE

---

② $x = 2\tan\theta \rightsquigarrow \tan\theta = \frac{x}{2} \Rightarrow$



$\Rightarrow \sec\theta = \frac{\sqrt{x^2+4}}{2}$

$dx = 2\sec^2\theta\, d\theta$

$\sqrt{x^2+4} = \sqrt{4\tan^2\theta + 4} = 2\sqrt{\tan^2\theta + 1} = 2\sec\theta \leftarrow \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\int \frac{dx}{\sqrt{x^2+4}} = \int \frac{2\sec^2\theta\, d\theta}{2\sec\theta} = \int \sec\theta\, d\theta = \ln|\sec\theta + \tan\theta| + c$

---

③ $\boxed{\begin{matrix} u = \cos x & dv = e^x dx \\ du = -\sin x dx & v = e^x \end{matrix}}$ $\boxed{\begin{matrix} \tilde{u} = \sin x & d\tilde{v} = e^x dx \\ d\tilde{u} = \cos x dx & \tilde{v} = e^x \end{matrix}}$

$\int e^x \cos^x dx \overset{u-dv}{=\!=} e^x \cos x - -\int e^x \sin x dx \overset{\tilde{u}-d\tilde{v}}{=\!=} e^x \cos x + e^x \sin x - \int e^x \cos x\, dx$

$\Rightarrow 2\int e^x \cos x\, dx = e^x(\cos x + \sin x) + K.$

---

⑤ $\frac{x^4 + 2x + 2}{x^4(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{D}{x^4} + \frac{E}{x+1} = \frac{Ax^3(x+1) + Bx^2(x+1) + Cx(x+1) + D(x+1) + Ex^4}{x^4(x+1)}$

$x^4 + 2x + 2 = Ax^3(x+1) + Bx^2(x+1) + Cx(x+1) + D(x+1) + Ex^4$ $\begin{matrix} \nearrow x = 0 \Rightarrow 2 = D \\ \searrow x = -1 \Rightarrow E = 1 \end{matrix}$

$x^4 \quad : 1 = A + E \to A = 0$

$x^3 \quad : 0 = A + B$

$x^2 \quad : 0 = B + C$ ⎤

$x \quad : 2 = C + D \to C = 0$ ⎦ $\to B = 0.$

Constant: $2 = D$

Note: $x^4 = (x-0)^4 = (\text{linear term})^4$

---

⑥ $x^3 + 1 \overline{)\,4x^3 - x + 1}$

$\quad\quad 4x^3 + 4$

$\quad\quad -x - 3$

and $(-1)^3 + 1 = 0. \Rightarrow x + 1$

$x+1 \overline{)\,x^3 \quad\quad +1}$ with quotient $x^2 - x + 1$:

$+x^2$

$-x^2 + 1$

$-x^2 - x$

$x + 1$

$x + 1$

0

$\frac{4x^3 - x + 1}{x^3 + 1} = 4 - \frac{x+3}{x^3+1}$

$\frac{x+3}{x^3+1} = \frac{x+3}{(x+1)(x^2 - x + 1)} = \frac{A}{x+1} + \frac{Bx + C}{x^2 - x + 1} = \frac{A(x^2 - x + 1) + (Bx + C)(x+1)}{(x+1)(x^2 - x + 1)}$

$\uparrow$ $b^2 - 4ac < 0$ $\quad x = -1 \Rightarrow 2 = A(1 + 1 + 1) \Rightarrow A = \frac{2}{3}$

$x^2 : 0 = A + B \to B = -\frac{2}{3}$

$x : 1 = -A + B + C$

Const : $3 = A + C \quad C = 3 - \frac{2}{3} = \frac{7}{3}$

$\int \frac{4x^3 - x + 1}{x^3 + 1}\, dx = \int 4dx - \frac{2}{3}\int \frac{dx}{x+1} + \frac{1}{3}\int \frac{2x - 7}{x^2 - x + 1}\, dx$

$= 4x - \frac{2}{3}\ln|x+1| + \frac{1}{3}\int \frac{2x - 1}{x^2 - x + 1}\, dx + \frac{1}{3}\int \frac{(-6)}{(x^2 - x + 1)}\, dx$

$= 4x - \frac{2}{3}\ln|x+1| + \frac{1}{3}\ln|x^2 - x + 1| - 2\int \frac{dx}{x^2 - x + 1}$

and

$\int \frac{dx}{x^2 - x + 1} = \int \frac{dx}{(x - \frac{1}{2})^2 + \frac{3}{4}} = \frac{\sqrt{3}}{2}\, \frac{4}{3} \int \frac{\sec^2\theta\, d\theta}{\sec^2\theta}$

$x - \frac{1}{2} = \frac{\sqrt{3}}{2}\tan\theta \to$ $\quad = \frac{2\sqrt{3}}{3}\int d\theta = \frac{2\sqrt{3}}{3}\theta + C$

$dx = \frac{\sqrt{3}}{2}\sec^2\theta\, d\theta$ $\quad = \frac{2\sqrt{3}}{3}\tan^{-1}\left(\frac{2}{\sqrt{3}}(x - \frac{1}{2})\right) + C$

$(x - \frac{1}{2})^2 + \frac{3}{4} = \frac{3}{4}\tan^2\theta + \frac{3}{4} = \frac{3}{4}\sec^2\theta$

---

8. of form ∞^0 so:

$y = x^{1/x} \Rightarrow \ln y = \ln(x^{1/x}) = \frac{1}{x}\ln x$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln x}{x} \overset{\cancel{\infty}}{\underset{L'H}{=}} \lim_{x\to\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to\infty} \frac{1}{x} = 0$

$\ln y \to 0$

$y = e^{\ln y} \to e^0 = 1.$