

MARK BOX

PROBLEM	POINTS
1	10
2	10
3	10
4	10
5	10
6	10
7	10
8	10
9	10
10	10
LSU vs USC	3
%	100

NAME: Librany

SSN: _____

Section 007 (MW 12:20 pm)

or

Section 008 (MW 2:30 pm)

INSTRUCTIONS:

- To receive credit you must:
 - work in a logical fashion, show all your work, indicate your reasoning
 - when applicable put your answer on/in the line/box provided
 - if no such line/box is provided, then box your answer
- The MARK BOX indicates the problems along with their points. Check that your copy of the exam has all of the problems.
- You may not use a calculator, books, personal notes. Give exact answers: for example, write $\ln 2$ instead of .6931, write $\sqrt{2}$ instead of 1.414, write π instead of 3.1415, write $\frac{1}{3}$ instead of 0.3333.
- During this exam, do not leave your seat. If you have a question, raise your hand. When you finish: turn your exam over, put your pencil down, and raise your hand.
- This exam covers (from *Calculus* by Varberg, Purcell, Rigdon, 8th ed.): Chapters 8 and 9.

Problem Inspiration:

- class handout of 119 integrals
- homework problem
- class handout of 119 integrals
- homework problem, an example from class, Girardi's Spring 03 exam 2
- an example from class
- class handout of 119 integrals, homework problem, an example from class
- homework problem
- homework problem, Girardi's Spring 03 exam 2
- homework problem
- an example from class homework problem

1.
$$\int \sec^3 x \tan^3 x dx = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

$$u = \tan x$$

$$du = \sec^2 x dx$$

or

$$u = \sec x$$

$$du = \sec x \tan x dx$$

$$\frac{\cos^2 x + \sin^2 x}{\cos^3 x} = \frac{1}{\cos^3 x}$$

$$\frac{1}{1 + \tan^2 x - \sec^2 x}$$

$$\int \sec^3 x \tan^3 x dx = \int \sec^2 x \tan^2 x (\sec x \tan x dx) = \int \sec^2 x (\sec^2 x - 1) (\sec x \tan x dx)$$

$$= \int u^2 (u^2 - 1) du = \int (u^4 - u^2) du = \frac{u^5}{5} - \frac{u^3}{3} + C$$

2.
$$\int \frac{x dx}{\sqrt{3x+4}} = \int \frac{u^2 - 4}{3} \cdot \frac{1}{3} \cdot \frac{2u du}{3} = \frac{2}{9} \left(\frac{u^3}{3} - 4u \right) = \frac{2(3x+4)^{3/2}}{27} - \frac{8(3x+4)^{1/2}}{9} + C$$

$$u = (3x+4)^{1/2} \Rightarrow u^2 = 3x+4 \Rightarrow 2udu = 3dx$$

3.
$$\int \frac{x^2 dx}{\sqrt{4-x^2}} = 2 \sin^{-1} \left(\frac{x}{2} \right) - 2 \cdot \frac{x}{2} \cdot \frac{\sqrt{4-x^2}}{2} + C \cong 2 \sin^{-1} \left(\frac{x}{2} \right) - \frac{x \sqrt{4-x^2}}{2} + C$$

$$x = 2 \sin \theta \rightarrow \sin \theta = \frac{x}{2}$$

$$dx = 2 \cos \theta d\theta$$

$$\sqrt{4-x^2} = \sqrt{4-4\sin^2 \theta} = 2 \sqrt{1-\sin^2 \theta} = 2 \sqrt{\cos^2 \theta} = 2 \cos \theta$$

$$\int \frac{x^2 dx}{\sqrt{4-x^2}} = \int \frac{(4\sin^2 \theta) (2\cos \theta d\theta)}{2\cos \theta} = 4 \int \sin^2 \theta d\theta = 4 \cdot \frac{1}{2} \int (1-\cos 2\theta) d\theta$$

$$= 2 \int d\theta - 2 \cdot \frac{1}{2} \int \cos 2\theta (2d\theta) = 2\theta - \sin 2\theta + C = 2\theta - 2\sin \theta \cos \theta + C$$

4.

$$\int e^{2x} \cos x dx = \frac{e^{2x} \sin x + 2e^{2x} \cos x}{5} + C$$

$$u = e^{2x} \quad dv = \cos x dx \quad \frac{du}{dx} = 2e^{2x} \quad \frac{dv}{dx} = \sin x$$

$$\int e^{2x} \cos x dx = e^{2x} \sin x - 2 \int e^{2x} \sin x dx$$

$$= e^{2x} \sin x - 2 \left[-e^{2x} \cos x - 2 \int e^{2x} \cos x dx \right]$$

$$= e^{2x} \sin x + 2e^{2x} \cos x - 4 \int e^{2x} \cos x dx$$

$$\Rightarrow 5 \int e^{2x} \cos x dx = e^{2x} \sin x + 2e^{2x} \cos x + C$$

5.

$$\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} dx = 2 \ln|x| + x^{-1} + \frac{3}{2} \ln|x^2 + 1| - 2 \arctan x + C$$

$$\frac{5x^3 - 3x^2 + 2x - 1}{x^2(x^2 + 1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 1} = \frac{A(x^2 + 1) + B(x^2 + 1) + (Cx + D)(x^2)}{x^2(x^2 + 1)}$$

$$\Rightarrow 5x^3 - 3x^2 + 2x - 1 = A(x^2 + 1) + B(x^2 + 1) + (Cx + D)(x^2) \rightarrow x=0 \Rightarrow -1=B$$

$$\Rightarrow x^3 : 5 = A + C \Rightarrow C=3$$

$$x^2 : -3 = B + D \Rightarrow D=-2$$

$$x : 2 = A \Rightarrow A=2$$

$$\text{Constant: } -1 = B$$

$$\int \frac{5x^3 - 3x^2 + 2x - 1}{x^4 + x^2} dx = 2 \int \frac{dx}{x} - 1 \int \frac{dx}{x^2} + 3 \int \frac{2x dx}{x^2 + 1} - 2 \int \frac{dx}{x^2 + 1}$$

6a. Let $n \in \mathbb{N} = \{1, 2, 3, 4, 5, \dots\}$. Derive a reduction formula for the below integral.

$$\int (\ln x)^n dx = x (\ln x)^n - n \int (\ln x)^{n-1} dx$$

HINT: Your solution should look like:

$\int (\ln x)^n dx = (\text{some function of } x) + (\text{maybe a constant}) \int (\ln x)^{\text{some number less than } n} dx$

$$u = (\ln x)^n \quad dv = dx$$

$$\frac{du}{dx} = n (\ln x)^{n-1} \cdot \frac{1}{x} \quad v = x$$

$$n=1 : \int \ln x dx = x (\ln x)^1 - 1 \int (\ln x)^0 dx = x \ln x - \int dx$$

$$n=2 : \int (\ln x)^2 dx = x (\ln x)^2 - 2 \int (\ln x)^1 dx$$

$$= x (\ln x)^2 - 2 (x \ln x - x) + C$$

6b. Use your solution to 6a to find the below 2 integrals.

$$\int \ln x dx = x \ln x - x + C$$

$$\int (\ln x)^2 dx = x (\ln x)^2 - 2x \ln x + 2x + C$$

7. if you L'Hopital lots of times (specifically 10000 times) you get

$$\lim_{x \rightarrow \infty} \frac{x^{10000}}{e^x} = \lim_{x \rightarrow \infty} \frac{\text{(some constant)}}{e^x} = 0$$

specifically

$$\lim_{x \rightarrow \infty} x^{\frac{1}{2}} = 1$$

of form ∞^0 so:

$$y = x^{\frac{1}{2}} \Rightarrow \ln y = \ln(x^{\frac{1}{2}}) = \frac{1}{2} \ln x$$

$$\lim_{x \rightarrow \infty} \ln y = \lim_{x \rightarrow \infty} \frac{\ln x}{x} \stackrel{\infty/\infty}{\sim} \lim_{x \rightarrow \infty} \frac{\frac{1}{x}}{1} = \lim_{x \rightarrow \infty} \frac{1}{x} = 0$$

$$\ln y \xrightarrow{x \rightarrow \infty} 0 \Rightarrow y = e^{\ln y} \xrightarrow{x \rightarrow \infty} e^0 = 1$$

$$\int_2^{\infty} \frac{\ln x dx}{x^2} = \frac{1 + \ln 2}{2}$$

$$u = \ln x \quad dv = x^{-2} dx = -\frac{\ln x}{x} + \int x^{-2} dx = -\frac{\ln x}{x} - \frac{1}{x} + C$$

$$du = \frac{dx}{x} \quad v = -x^{-1}$$

$$\int_2^{\infty} \frac{\ln x dx}{x^2} = \lim_{b \rightarrow \infty} \int_2^b \frac{\ln x dx}{x^2} = \lim_{b \rightarrow \infty} \left[-\frac{\ln x}{x} - \frac{1}{x} \right]_{x=2}^{x=b}$$

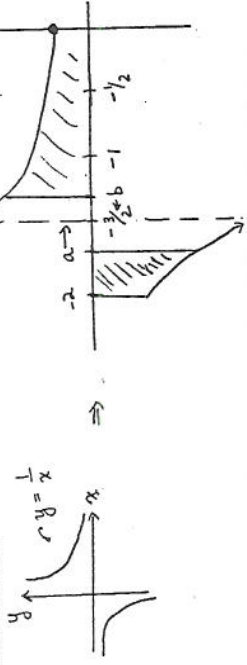
$$= \lim_{b \rightarrow \infty} \left[\frac{1 + \ln x}{x} \Big|_{x=b}^{x=2} \right] = \lim_{b \rightarrow \infty} \left[\frac{1 + \ln 2}{2} + \frac{1 + \ln b}{b} \right]$$

$$= \frac{1 + \ln 2}{2} + \lim_{b \rightarrow \infty} \frac{1 + \ln b}{b} \stackrel{L'H}{=} \frac{1 + \ln 2}{2} + \lim_{b \rightarrow \infty} \frac{1/b}{1} = 0$$

10.

$$\int_{-2}^0 \frac{dx}{2x+3} = \text{DNE or diverges.}$$

HINT: make a (very) rough sketch of the integrand (i.e., the function you need to integrate - where is it not defined).



$$\int_{-2}^0 \frac{dx}{2x+3} = \lim_{a \rightarrow -3/2^-} \frac{1}{2} \int_{-2}^a \frac{2 dx}{2x+3} + \lim_{b \rightarrow -3/2^+} \frac{1}{2} \int_b^0 \frac{2 dx}{2x+3}$$

$$= \lim_{a \rightarrow -3/2^-} \frac{1}{2} \ln|2a+3| \Big|_{x=-2}^{x=a} + \lim_{b \rightarrow -3/2^+} \frac{1}{2} \ln|2b+3| \Big|_{x=b}^{x=0}$$

$$= \left[\lim_{a \rightarrow -3/2^-} \frac{\ln|2a+3|}{2} \right] - \frac{\ln 1}{2} + \frac{\ln 3}{2} - \lim_{b \rightarrow -3/2^+} \frac{\ln|2b+3|}{2}$$

$$= -\infty - 0 + \frac{\ln 3}{2} - \infty = -\infty$$