1. Let $f(x) = x^2 + 2x$ for $x \le -1$.

1a. Find the inverse function of $y = f(x)$. Answer: $f^{-1}(x) =$ ______

Let $y = f^{-1}(x)$. So $f(y) = x$ and $y \le -1$. It follows that $y^2 + 2y = x$. I could use the quadratic formula. I will just complete the square.

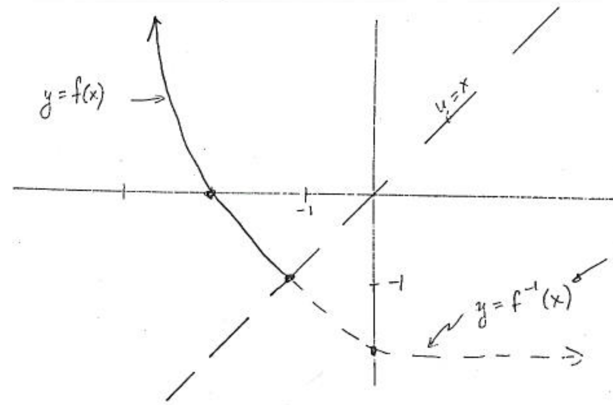
$$
\begin{aligned}
(y+1)^2 &= x+1 \\
y+1 &= \pm\sqrt{x+1} \\
y &= -1 \pm \sqrt{x+1}.
\end{aligned}
$$

But $y \le -1$; so, $y = -1 - \sqrt{x+1}$. We conclude that

$$\boxed{f^{-1}(x) = -1 - \sqrt{x+1}}$$

1b. Sketch the graph of $y = f(x)$ and $y = f^{-1}(x)$ on the same grid. Explain how you got the graph of $y = f^{-1}(x)$ from the graph of $y = f(x)$.

$f(x) = x^2 + 2x = (x)(x - -2) \Rightarrow$ x-intercepts at 0 and -2 $\Rightarrow$ vertex at $(-1, -$



to obtain this graph, reflect graph of $y = f(x)$ over the line $y = x$

1c. The domain of $y = f^{-1}(x)$ is $x \ge -1$ or $[-1, \infty)$

2. Solve the equation

$$\log_2(x+4) - \log_2 x = 3$$

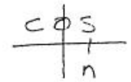
for x. Your answer should **not** have a logarithm nor exponential in it.

ANSWER: $x = \dfrac{4}{7}$

$\log_2\left(\frac{x+4}{x}\right) = 3 \Rightarrow \frac{x+4}{x} = 2^3 \Rightarrow 1 + \frac{4}{x} = 8 \Rightarrow \frac{4}{x} = 7$

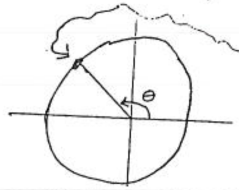
3. Simplify $\sin\left(2\arccos\left(-\frac{3}{5}\right)\right)$. Your answer should be a rational number.

ANSWER: $\sin\left(2\arccos\left(-\frac{3}{5}\right)\right) = 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{-24}{25}$



For this test I'll remind you that $\sin(2\theta) = 2\sin\theta\cos\theta$ but you better know this by the next test!

Let $\theta = \arccos\left(-\frac{3}{5}\right) \Rightarrow \cos\theta = -3/5$



$= (\cos\theta, \sin\theta) = \left(-\frac{3}{5}, +\sqrt{1 - \left(\frac{3}{5}\right)^2}\right)$

$= \left(-\frac{3}{5}, \sqrt{\frac{25-9}{25}}\right) = \left(-\frac{3}{5}, \frac{4}{5}\right)$

4a. $D_x\left[(3x+1)^2\right] = 2(3x+1)^1 \, 3 = 6(3x+1)$

4b. $D_x\left[2^{(3x+1)}\right] = \left(2^{3x+1}\right)(\ln 2)(3)$

4c. $D_x\left[(3x+1)^x\right] = (3x+1)^x\left[\ln(3x+1) + \frac{3x}{3x+1}\right]$

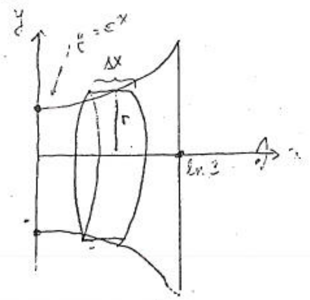
$y = (3x+1)^x \Rightarrow \ln y = x\ln(3x+1) \Rightarrow \frac{1}{y}\frac{dy}{dx} = \ln(3x+1) + x\,\frac{1}{3x+1}\,(3)$

8. The volume of the solid generated by revolving the region bounded by:

$y = e^x$ and the x-axis and the y-axis and $x = \ln 3$

about the x-axis is $\boxed{\frac{8\pi}{2} = 4\pi}$.

Your answer should **not** have a logarithm nor exponential in it.



$V_{typical} = (\text{area of base})(\text{height}) = \pi [f(x)]^2 \Delta x$

$V = \pi \int_{x=0}^{x=\ln 3} [e^x]^2 \, dx = \pi \int_{x=0}^{x=\ln 3} e^{2x} \, dx$

$= \frac{\pi}{2} e^{2x} \Big|_{x=0}^{x=\ln 3} = \frac{\pi}{2}\left[e^{2\ln 3} - e^0\right]$

$= \frac{\pi}{2}\left[e^{\ln 3^2} - 1\right] = \frac{\pi}{2}[9-1]$

5. $D_x\,[\,3\ln(1+e^{5x})\,] = \dfrac{3\,e^{5x}(5)}{1+e^{5x}} = \dfrac{15\,e^{5x}}{1+e^{5x}}$

6. $\int \frac{2\ln x}{x}\,dx = 2\int \ln x \left(\frac{dx}{x}\right) = 2\int u\,du = u^2 + C = \boxed{(\ln x)^2} + C$

$u = \ln x$
$du = \frac{dx}{x}$

7. $\int e^{(3x+1)}\,dx = \frac{1}{3}\int e^u\,du = \frac{1}{3}e^u + C = \boxed{\frac{1}{3}e^{3x+1}} + C$

$u = 3x+1$
$du = 3dx$

9. The solution to the differential equation

$$\frac{dy}{dt} = -6\,y(t)$$

subject to the condition that $y(2) = 7$ is $\boxed{y(t) = \dfrac{7e^{-6t}}{e^{-12}}}$.

You may assume that the $y(t) \geq 0$ for each t.

$\frac{1}{y(t)}\frac{dy}{dt} = -6 \Rightarrow D_t \ln y(t) = D_t(-6t) \Rightarrow \ln y(t) = -6t + C$

$\Rightarrow y(t) = e^{-6t+C} \Rightarrow \boxed{y(t) = e^{C}e^{-6t}}$ ← Almost done...

$\Rightarrow y(2) = 7 = e^{C}e^{-6(2)} \Rightarrow e^{C} = 7/e^{-12}$

$\Rightarrow \frac{dP}{dt} = k\,P(t) \Rightarrow P(t) = P_0 e^{kt}$

10. The rate of decay of a radioactive substance is proportional to the amount of such substance present. Today we have 10 grams of a radioactive substance. Given that one-third of the substance decays every 8 years, how much will be left t years from today? *Clearly explain your notation.*

ANSWER: $P(t) = 10\exp\left(\frac{t\ln(2/3)}{8}\right)$ grams

$\frac{1}{3}$ decays

$\frac{2}{3}$ r

Let t_0 = initial time = today

t = the # of years after t_0

$P(t)$ = the # of grams of the radioactive substance at time t.

$P(t) = 10e^{kt}$ ← now to find k

$P(8) = 10e^{8k} = \frac{2}{3}\cdot 10 \Rightarrow e^{8k} = \frac{2}{3} \Rightarrow 8k = \ln\left(\frac{2}{3}\right) \Rightarrow k =$