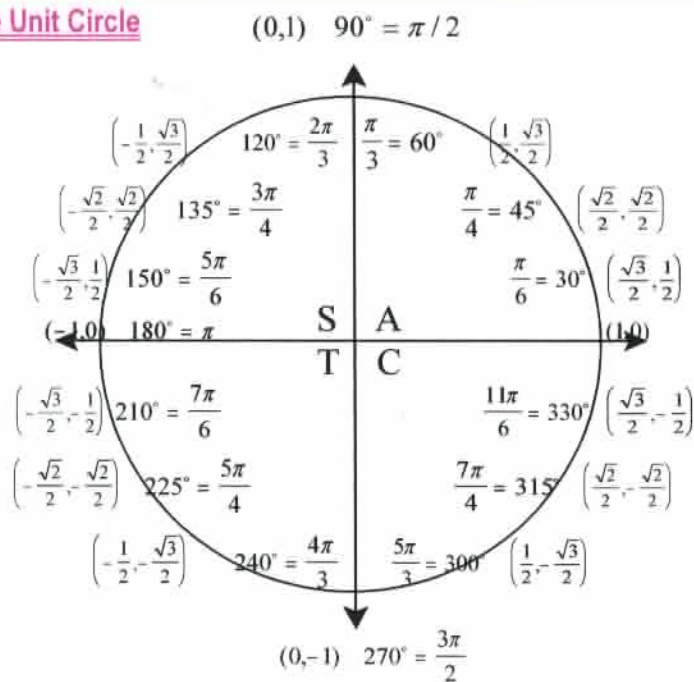


TRIGONOMETRY

The Unit Circle

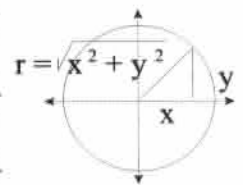


Circular Function Definitions:

$$\sin \theta = \frac{y}{r} \quad \csc \theta = \frac{r}{y}$$

$$\cos \theta = \frac{x}{r} \quad \sec \theta = \frac{r}{x}$$

$$\tan \theta = \frac{y}{x} \quad \cot \theta = \frac{x}{y}$$



Right Triangle Definitions:

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \csc \theta = \frac{\text{hyp}}{\text{opp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \sec \theta = \frac{\text{hyp}}{\text{adj}}$$

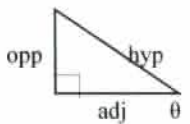
$$\tan \theta = \frac{\text{opp}}{\text{adj}} \quad \cot \theta = \frac{\text{adj}}{\text{opp}}$$

Odd Functions

$\sin \theta, \csc \theta,$
 $\tan \theta, \cot \theta$

Even Functions

$\cos \theta, \sec \theta$



Reciprocal Identities

$$\sin \theta = \frac{1}{\csc \theta} \quad \csc \theta = \frac{1}{\sin \theta}$$

$$\cos \theta = \frac{1}{\sec \theta} \quad \sec \theta = \frac{1}{\cos \theta}$$

$$\tan \theta = \frac{1}{\cot \theta} \quad \cot \theta = \frac{1}{\tan \theta}$$

Negative Angle Identities

$$\sin(-\theta) = -\sin \theta \quad \csc(-\theta) = -\csc \theta$$

$$\cos(-\theta) = \cos \theta \quad \sec(-\theta) = \sec \theta$$

$$\tan(-\theta) = -\tan \theta \quad \cot(-\theta) = -\cot \theta$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} [\cos(u-v) - \cos(u+v)]$$

$$\cos u \cos v = \frac{1}{2} [\cos(u-v) + \cos(u+v)]$$

$$\sin u \cos v = \frac{1}{2} [\sin(u+v) + \sin(u-v)]$$

$$\cos u \sin v = \frac{1}{2} [\sin(u+v) - \sin(u-v)]$$

Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \quad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

Double-Angle Formulas

$$\sin 2u = 2 \sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2 \cos^2 u - 1$$

$$= 1 - 2 \sin^2 u$$

$$\tan 2u = \frac{2 \tan u}{1 - \tan^2 u}$$

Power-Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Cofunction Identities

$$\sin(90^\circ - \theta) = \cos \theta \quad \cos(90^\circ - \theta) = \sin \theta$$

$$\tan(90^\circ - \theta) = \cot \theta \quad \cot(90^\circ - \theta) = \tan \theta$$

$$\sec(90^\circ - \theta) = \csc \theta \quad \csc(90^\circ - \theta) = \sec \theta$$

Sum & Difference Formulas

$$\sin(u \pm v) = \sin u \cos v \pm \cos u \sin v$$

$$\cos(u \pm v) = \cos u \cos v \mp \sin u \sin v$$

$$\tan(u \pm v) = \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v}$$

Sum-to-Product Formulas

$$\sin u + \sin v = 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\sin u - \sin v = 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

$$\cos u + \cos v = 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right)$$

$$\cos u - \cos v = -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right)$$

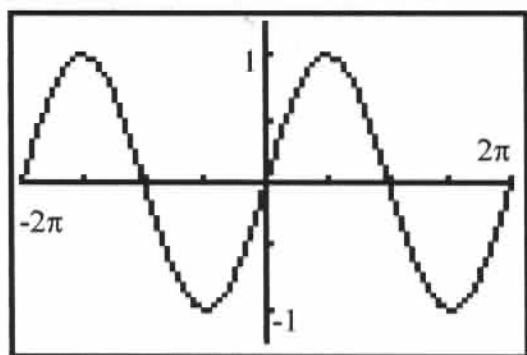
Half-Angle Formulas

$$\sin\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 - \cos u}{2}} \quad \cos\left(\frac{u}{2}\right) = \pm \sqrt{\frac{1 + \cos u}{2}} \quad \tan\left(\frac{u}{2}\right) = \frac{1 - \cos u}{\sin u} = \frac{\sin u}{1 + \cos u}$$

Graphs of the Trigonometric Functions

$y = \sin x$

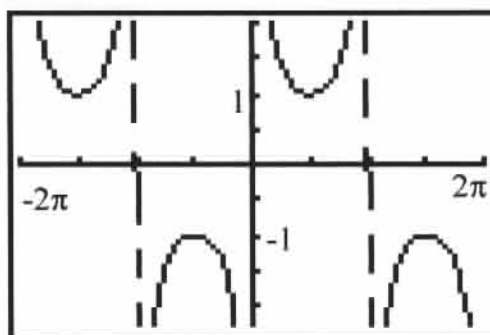
Symmetry: Odd



Domain: $(-\infty, \infty)$
 Range: $[-1, 1]$
 Period: 2π

$y = \csc x$

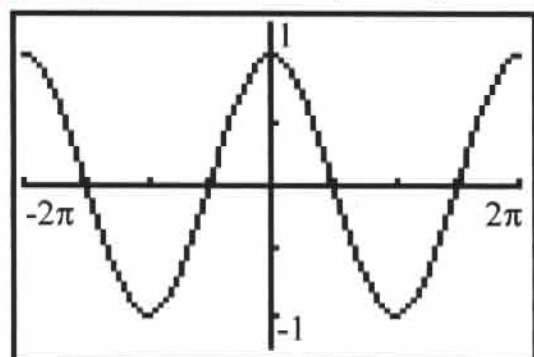
Symmetry: Odd



Domain: all reals except $x = \pi n$ where $n = \text{integer}$
 Range: $(-\infty, -1], [1, \infty)$ Asymptotes: πn
 Period: 2π

$y = \cos x$

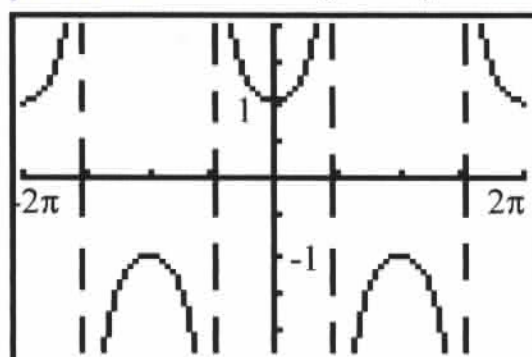
Symmetry: Even



Domain: $(-\infty, \infty)$
 Range: $[-1, 1]$
 Period: 2π

$y = \sec x$

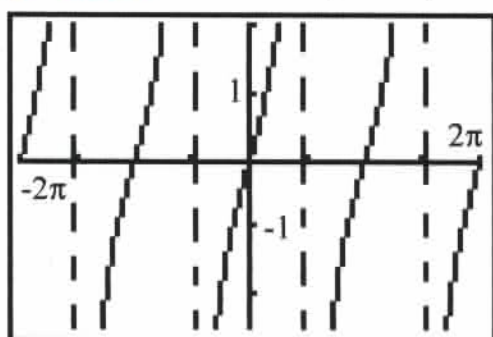
Symmetry: Even



Domain: all reals except $x = \pi/2 + \pi n$ where $n = \text{integer}$
 Range: $(-\infty, -1], [1, \infty)$ Asymptotes: $\pi/2 + \pi n$
 Period: 2π

$y = \tan x$

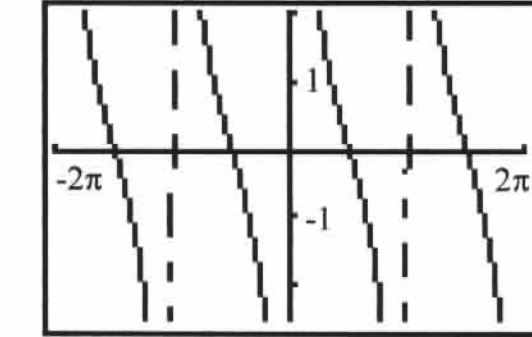
Symmetry: Odd



Domain: all reals except $x = \pi/2 + \pi n$ where $n = \text{integer}$
 Range: $(-\infty, \infty)$ Asymptotes: $\pi/2 + \pi n$
 Period: π

$y = \cot x$

Symmetry: Odd



Domain: all reals except $x = \pi n$ where $n = \text{integer}$
 Range: $(-\infty, \infty)$ Asymptotes: πn
 Period: π