Definitions & Theorems

Asymptotes:

V.A.: denominator = 0

$lim_{x \to c^+} f(x) = \frac{1}{0}$ $lim_{x \to c^-} f(x) = \frac{1}{0}$ * VA at x = c

Use your "brain" to determine whether the limit is approaching infinity or negative infinity by substituting a value close to *c* into the function. If it is a positive value, the limit = ∞. If it is a negative value, the limit = - ∞.

H.A.: degree of the numerator < degree of the denominator… y = 0 $lim_{x \to \pm\infty} f(x) = 0$
degree of the numerator = degree of the denominator… y = leading coeffs $lim_{x \to \pm\infty} f(x) = H.A.$
degree of the numerator > degree of the denominator… long division $lim_{x \to \pm\infty} f(x) = \infty$
or $lim_{x \to \pm\infty} f(x) = -\infty$

Limits:

* A hole does not prevent a limit from existing.
* The limit of a constant is a constant.
* If the limit is approaching ∞ or - ∞, think about H.A. or divide every term by the variable with the largest degree in the denominator. Then, remember that: $lim_{x \to \pm\infty} \frac{1}{x} = 0$.
* If the answer is of the form $lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$, we have an indeterminate form. Try one of the following techniques:
  - Factor the numerator & denominator, simplify, and then substitute the value of *c* into the new expression.
  - Rationalize the numerator or the denominator by multiplying by the conjugate.
  - Simplify the complex fraction by getting a common denominator.

* Special Limits: $lim_{x \to 0} \frac{\sin(ax)}{ax} = 1$ $lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

* Piecewise-Defined Functions
  - To find the limit of the function as x approaches a value where the function does not split: Evaluate the limit of the function for that specific piece of the function.
  - To find the limit of the function as x approaches a value where the function does split: Evaluate a left-hand limit using the piece of function on the left side & evaluate a right-hand limit using the piece of function on the right side. If the left-hand limit = the right-hand limit, then an overall limit exists.

Intermediate Value Theorem:

A function that is continuous on a closed interval $[a, b]$ takes on every value between $f(a)$ and $f(b)$. In other words, if y_0 is between $f(a)$ and $f(b)$, then $y_0 = c$ for some c in $[a, b]$.

Continuity:

1) $f(c)$ must exist … the point must exist
2) $lim_{x \to c} f(x)$ must exist
3) $lim_{x \to c} f(x) = f(c)$

* Endpoints only need one-sided limits to prove that the function is continuous at that point.

Average Rate of Change vs. Instantaneous Rate of Change:

Average Rate of Change = slope of the secant line

$$m = \frac{f(b)-f(a)}{b-a} = \frac{f(x+h)-f(x)}{h}$$

Instantaneous Rate of Change = slope of the tangent line (derivative)

$$m = lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

* Algebraically: original denominator of *h* must cancel.

Derivative:

* Rate of change (instantaneous) = slope of the function = slope of the tangent line
* The differentiable function must be continuous. But, a continuous function doesn't have to be differentiable. Think about the graph of the absolute value of *x*.
* The derivative of a constant is zero.
* If a function has a derivative it is said to be differentiable.

Derivative of an Inverse Function:

If *f* & *g* are inverses of each other, the derivative of *f* can be found as follows: $f'(x) = \frac{1}{g'(f(x))}$

* The derivative of a function = 1/(derivative of the inverse)
* Twist: What value goes into the inverse? Not the value given for the function!

Implicit Differentiation:

When you can't isolate *y* in terms of *x*, it's time to take the derivative implicitly.

1) Differentiate both sides of the equation with respect to *x*. Any time you take a derivative of y label it *dy*/*dx*.
2) Collect the terms with *dy*/*dx* on one side of the equation.
3) Factor out *dy*/*dx*.
4) Solve for *dy*/*dx* by dividing.

Normal Line:

A normal line is simply the line perpendicular to the tangent line at the same point. Therefore the slopes are negative reciprocals of each other.

Linearization:

Finding the equation for the tangent line.

Differentials:

$dy = f'(x_0)dx$

Take the derivative, *dy*/*dx*, & multiply *dx* to the other side of the equation
dy: approximate value of *y* between the tangent line & value of the function
dx: the change in the values of the *x*'s

$f(x) = dy + f(x_0)$

Critical Points:

Points on the graph of a function where the derivative is zero or is undefined.

$f'(x) = \frac{g(x)}{h(x)}$ * Derivative = 0 when $g(x) = 0$. (Numerator = 0)

* Derivative is undefined when $h(x) = 0$. (Denominator = 0)

	maximum on f	minimum on f	increasing on f	decreasing on f	concave up on f	concave down on f	point of inflection
f original function	interior points or endpoints	interior points or endpoint	positive slope	negative slope	"cup" up	"cup" down	interior point between c-up & c-down
f' original's first derivative	$f'=0$ changes + to -	$f'=0$ changes - to +	$f'>0$	$f'<0$	f' is increasing	f' is decreasing	max/ min
f'' original's second derivative	$f'(c)=0$ and $f''(c)<0$ maximum point at $x=c$	$f'(c)=0$ and $f''(c)>0$ minimum point at $x=c$			$f''>0$	$f''<0$	$f''=0$ changes - to + or + to − No endpoints

Particle Motion:

* Position of the particle at time t, usually denoted $x(t)$ or $s(t)$, is its location on the number line if it moves in one dimension, or its height if it moves in two dimensions.

* Velocity $v(t) = s'(t)$ of the particle is the rate of change of position with respect to time, or the derivative of position.

- $v(t) = 0$ Particle at rest
- $v(t) > 0$ Particle moves to right (or up)
- $v(t) < 0$ Particle moves to left (or down)
- Sign of $v(t)$ changes Particle changes direction
- $|v(t)|$ Speed is the absolute value of velocity

* Acceleration $a(t) = v'(t) = s''(t)$ of the particle is the rate of change of velocity with respect to time, or the derivative of velocity.

* When the velocity and acceleration of the particle have the same signs, the particle's speed is increasing.
* When the velocity and acceleration of the particle have the opposite signs, the particle's speed is decreasing (or slowing down).

Newton's Method:

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

Calculator: Enter the function into y_1 and derivative into y_2

Homescreen: Enter initial guess [sto->] as "x"

Enter: x – y_1/y_2 [sto->] x

Keep on hitting enter until the values satisfy the desire of accuracy (4 places)

Mean Value Theorem:

If a function is continuous on the closed interval [*a*, *b*] & differentiable on the open interval (*a*, *b*), then

$f'(c) = \frac{f(b)-f(a)}{b-a}$

* Instantaneous rate of change = Average rate of change
* Slope of the tangent line = Slope of the secant line

Rolle's Theorem:

If a function is continuous on the closed interval [*a*, *b*], differentiable on the open interval (*a*, *b*) and has the same y-value at the endpoints *a* and *b*, then there must be at least one value of *x*, call it *c*, between *a* and *b* where the function has a horizontal tangent (max or min point).

Riemann Sums: RRAM, MRAM, LRAM:

The left-hand, right-hand, and midpoint sums are all examples of a more general approach to finding areas called Riemann sums. In a Riemann sum, a function & an interval are given, the interval is partitioned, and the height of each rectangle can be the value of the function at any point in the subinterval. In fact, even the subintervals do not necessarily need to be the same length. All you need to remember is that a Riemann sum uses rectangles to find the area between a curve and the x-axis.

Area Under a Curve:

* An integral will calculate the area under the curve to the x-axis.
* If the area is under the x-axis, the integral will be equal to a negative value.

Area Between Two Curves:

* Vertical rectangles: Top – Bottom
* Horizontal rectangles: Right - Left

Trapezoidal Rule:

$Trap = \frac{b-a}{2n}(y_0 + 2y_1 + 2y_2 + \cdots + 2y_{n-1} + y_n)$

Antiderivatives:

* The process of finding an antiderivative is called antidifferentiation or integration.

Fundamental Theorem of Calculus, Part 1 & 2:

$F(x) = \int_a^x f(t)dt$

$\frac{dF}{dx} = \frac{d}{dx}\int_a^x f(t)dt = f(x)$ ⟶ $\frac{dF}{dx} = \frac{d}{dx}\int_a^x \frac{1}{t+2}dt = \frac{1}{x+2}$

$\frac{dF}{dx} = \frac{d}{dx}\int_a^{v(x)} f(t)dt = f(v(x))v'(x)$ ⟶ $\frac{dF}{dx} = \frac{d}{dx}\int_a^{x^3} \sec(t)dt = 3x^2\sec(x^3)$

$\int_a^b f(t)dt = F(b) - F(a)$

$\int_b^a f(t)dt = F(a) - F(b) = -[F(b) - F(a)]$

$\int_a^a f(t)dt = 0$

Integrals:

* Total Distance Traveled or Total Area

ABS value

1) Find out if the function goes below the x-axis.
2) Take the absolute value of the areas… no negative values $\int_a^b |f(x)|dx$

Math num. 1

* Position Shift or Displacement = integral: $\int_b^a f(x)dx$ (Keep negative values)

* Maximum value, minimum value or y-value: Displacement + Initial value
(Is there an initial value given?)

Average Value:

$\frac{1}{b-a}\int_a^b f(x)dx$

Volumes:

Disk Method: $\pi\int_a^b [f(x)]^2 dx$ * Revolve around x-axis integrate w.r.t. x's

Washer Method: $\pi\int_a^b [(R(x))^2 - (r(x))^2]dx$ * Revolve around y-axis integrate w.r.t. y's.

Shell Method: $2\pi\int_a^b (radius)(height)dx$
* Height: deals with the functions: (top – bottom) or (right – left)
* Radius: deals with axis of rotation & x variable (if revolved about y's) or axis of rotation & y variable (if revolved about x's)

Slicing:
1) If cross-section is perpendicular to x-axis integrate w.r.t. x's or if it is perpendicular to y-axis integrate w.r.t. y's.
2) What is the shape of the cross-section? Find the area formula.
3) Incorporate the functions into the area formula.
4) Integrate

L'Hopital's Rule:

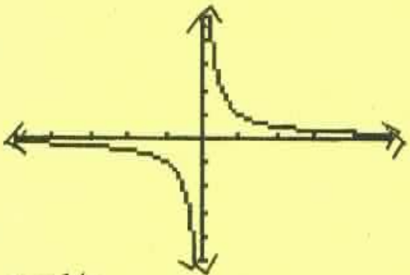
$$lim_{x\to c}\frac{f(x)}{g(x)} = \frac{f'(c)}{g'(c)}$$

* Apply only when taking the limit of a quotient. If the expression is an indeterminate form, but not in the form of a quotient, it must be rewritten in quotient form.
* Remember to take the derivative of the numerator & the derivative of the denominator. Do not use the Quotient Rule!
* Be sure to recheck each time L'Hopital's Rule is applied to see if the result is still indeterminate. It may be applied more than once, but only if the limit results in an indeterminate form each time.

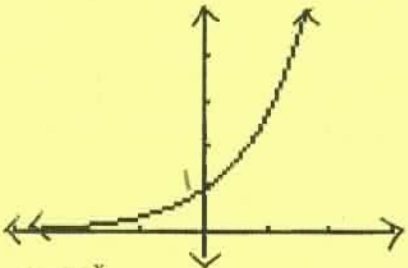
Euler's Method:

$$y_{n+1} = y_n + hf(x_0, y_0)$$

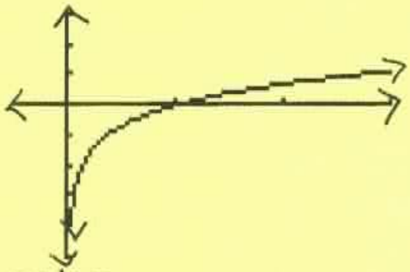
Graphs:



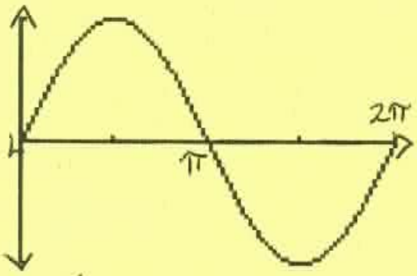
y = 1/x
Horizontal Asymptote at y = 0
Vertical Asymptote at x = 0



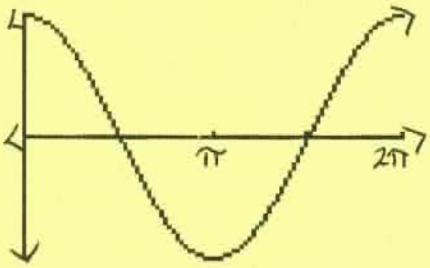
y = e ^{x}
Horizontal Asymptote at y = 0



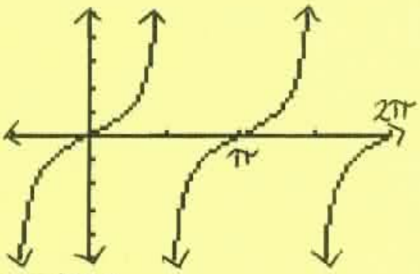
y = ln x
Vertical Asymptote at x = 0



y = sin x
Period: 2π



y = cos x
Period: 2π



y = tan x
Period: π
Asymptotes at x = $\frac{\pi}{2}k$

Pythagorean Identities:

$sin^2x + cos^2x = 1$ $\quad$ $1 + tan^2x = sec^2x$ $\quad$ $1 + cot^2x = csc^2x$

Power-Reduction Identities:

$sin^2x = \frac{1-cos2x}{2}$ $\quad$ $cos^2x = \frac{1+cos2x}{2}$

Inverse Trigonometric Functions:

$y = \sin^{-1} x$ Quad: I, IV
$= \csc^{-1}(1/x)$

$y = \cos^{-1} x$ Quad: I, II
$= \sec^{-1}(1/x)$

$y = \tan^{-1} x$ Quad: I, IV

Differentiation Formulas:

$\frac{d}{dx}(sinu) = cosu\frac{du}{dx}$

$\frac{d}{dx}(cosu) = -sinu\frac{du}{dx}$

$\frac{d}{dx}(tanu) = sec^2u\frac{du}{dx}$

$\frac{d}{dx}(cotu) = -csc^2u\frac{du}{dx}$

$\frac{d}{dx}(secu) = secutanu\frac{du}{dx}$

$\frac{d}{dx}(cscu) = -cscucotu\frac{du}{dx}$

$\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$

$\frac{d}{dx}(a^u) = a^u lna\frac{du}{dx}$

$\frac{d}{dx}(lnu) = \frac{1}{u}\frac{du}{dx}$

$\frac{d}{dx}(log_a u) = \frac{1}{lna}\frac{1}{u}\frac{du}{dx}$ or use the Change of Base formula

$log_a u = \frac{lnu}{lna}$

$\frac{d}{dx}(sin^{-1}u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}(tan^{-1}u) = \frac{1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}(sec^{-1}u) = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

$\frac{d}{dx}(cos^{-1}u) = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx}$

$\frac{d}{dx}(cot^{-1}u) = \frac{-1}{1+u^2}\frac{du}{dx}$

$\frac{d}{dx}(csc^{-1}u) = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx}$

Integration Formulas:

$\int(sinu)du = -cosu + C$

$\int(cosu)du = sinu + C$

$\int(sec^2u)du = tanu + C$

$\int(csc^2u)du = -cotu + C$

$\int(secutanu)du = secu + C$

$\int(cscucotu)du = -cscu + C$

$\int e^u\,du = e^u + C$

$\int a^u\,du = \frac{a^u}{lna} + C$

$\int \frac{1}{u}du = ln|u| + C$

$\int(tanu)du = -ln|cosu| + C = ln|secu| + C$

$\int(cotu)du = ln|sinu| + C = -ln|cscu| + C$

$\int (secu)du = ln|secu + tanu| + C$

$\int (cscu)du = -ln|cscu + cotu| + C$

$\int \frac{du}{\sqrt{a^2-u^2}} = sin^{-1}\left(\frac{u}{a}\right) + C$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}sec^{-1}\left|\frac{u}{a}\right| + C = \frac{1}{a}cos^{-1}\left|\frac{a}{u}\right| + C$

$\int \frac{du}{u^2+a^2} = \frac{1}{a}tan^{-1}\left(\frac{u}{a}\right) + C$

<u>Methods Needed to Find the Following Integrals:</u>

$\int (sin^2u)du = \int \frac{1-cos2u}{2}du$	Power-Reducing Identity
$\int (cos^2u)du = \int \frac{1+cos2u}{2}du$	Power-Reducing Identity
$\int tan^2udu = \int (sec^2u - 1)\,du$	Pythagorean Identity
$\int cot^2udu = \int (csc^2u - 1)\,du$	Pythagorean Identity
$\int (log_a u)du = \int \frac{lnu}{lna}du$	Change of Base Formula