

26 June 2007
Math 242

More Exercises on Laplace Transforms

Directions: Please complete the following problems for homework. This will be due on Monday, July 2.

1. Complete the worksheet that I handed out on Laplace transforms. There is no need to turn this in but you should try to do as many of these as possible for extra practice. The solutions are on the back of the worksheet.
2. Solve the following initial value problems using Laplace transforms:

$$\begin{cases} y' + 6y = 4t \\ y(0) = 2 \end{cases} \quad \begin{cases} y''' + 2y'' - y' - 2y = \sin(3t) \\ y(0) = 0 \\ y'(0) = 0 \\ y''(0) = 1 \end{cases} \quad \begin{cases} y'' - 5y' + 6y = H(t - 1) \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

3. Consider the function $f(t) = \begin{cases} 0, & 0 \leq t < \pi \\ 1, & \pi \leq t < 2\pi \\ 0, & t \geq 2\pi \end{cases}$

- (a) Write $f(t)$ in terms of the Heaviside function.
- (b) Solve the initial value problem

$$\begin{cases} y'' + y = f(t) \\ y(0) = 0 \\ y'(0) = 1 \end{cases}$$

4. Consider the function $f(t) = \begin{cases} 1, & 0 \leq t < \pi/2 \\ \sin t, & t \geq \pi/2 \end{cases}$

(a) Write $f(t)$ in terms of the Heaviside function.

(b) Solve the initial value problem

$$\begin{cases} y'' + y = f(t) \\ y(0) = 1 \\ y'(0) = 0 \end{cases}$$

5. An *integral equation* is an equation involving an unknown function $f(t)$ and various integrals thereof. Sometimes, Laplace transforms can enable us to solve integral equations.

(a) Verify that $f(t) = \sin t$ solves the integral equation

$$f(t) + \int_0^t (t - \tau)f(\tau)d\tau = t$$

(b) Take the Laplace transform of both sides of the integral equation above and then solve for $F(s) = \mathcal{L}\{f(t)\}$. Now, take the inverse transform of $F(s)$. You should get the answer given to you in part (a).