Math 242
Summer I 2007

Homework 4 (Second Order Equations)

Please work the following problems for homework. This will be due on Monday, June 25 during class. This is quite a bit of work so be sure to start as soon as possible.

1. Determine whether or not the following sets of functions are linearly independent or not.

   (a) $\cos x \sin x$, $\sin(2x)$

   (b) e^{2x}, e^{-x}

   (c) $1, x, x^2$

2. Find a fundamental set for the following homogenous equations and then write down the homogenous solution. For (e)-(f), first try to find a root of the auxillary equation by inspection and then perform polynomial division. Then, for parts (a)-(c), solve the IVP.

   (a) $y'' + 3y = 0$ constrained to $y(0) = 0$ and $y'(0) = 2\sqrt{3}$.

   (b) $y'' + 3y' - 10y = 0$ constrained to $y(0) = 0$ and $y'(0) = 1$.

   (c) $y'' - 6y' + 9y = 0$ constrained to $y(0) = 2$ and $y'(0) = 10$.

   (d) $y'' - 4y' + 13y = 0$

   (e) $y''' + 4y'' + y' - 6y = 0$

   (f) $y''' - 9y'' + 24y' - 20y = 0$

3. Solve the following non-homogenous problems using the method of undetermined coeficients. Write down the general solution.

   (a) $y'' + 4y = 2xe^{-x}$

   (b) $y'' - 6y' + 8y = x^2 + 1$

   (c) $y'' + y' - 2y = \sin(3x)$

(d) $y'' + y' - 2y = e^x$

(e) $y''' + 4y'' + y' - 6y = 2$ (you found the homogenous solution to this in part 2e).

4. Solve the following non-homogenous problems using the method of variation of parameters. Write down the general solution.

   (a) $y'' - 2y' + y = \frac{e^x}{x^2+1}$

   (b) $y'' + y = \sec x$

5. Consider the initial value problem

$$\begin{cases} xy'' - y' + 4x^3y = 2x^3 \\ y(\sqrt{\pi}) = -\frac{1}{2} \\ y'(\sqrt{\pi}) = 4\sqrt{\pi} \end{cases}$$

   (a) Verify that $y_1(x) = \sin(x^2)$ is a solution to $xy'' - y' + 4x^3y = 0$.

   (b) Use the fact that $y_1(x) = \sin(x^2)$ solves the homogenous problem in (a) to find a fundamental set for the homogenous problem.

   (c) Use variation of parameters and the fundamental set you obtained in (b) to solve the nonhomogenous problem above.

   (d) Write down the general solution and solve the initial value problem.

6. Solve the following IVP:

$$\begin{cases} 4x^2y'' + 8xy' + y = \ln x \\ y(1) = 2 \\ y'(1) = 0 \end{cases}$$