23 April 2008
Math 242

Information on Final Exam

1 General Information

The final exam will be comprehensive; anything that we covered during the course of the semester is fair game for the final. It will be held on Monday, May 5 at 9:00AM in our usual classroom. I will allow you to bring the following items to the final: (a) an $8\frac{1}{2} \times 11$ inch sheet of paper containing formulae or anything else you see fit to write (you may utilize both sides if you so desire) and (b) a standard scientific calculator. I will not allow you to use a graphing calculator or anything sufficiently advanced (like a TI-89).

2 Study Tips

I am not asking you to memorize a plethora of formulae; that is the primary reason I am allowing you to bring a 'cheat sheet' so to speak. I am more interested in your ability to use the formulae. Therefore, here are some tips that might be helpful during your study time:

(a) When studying a particular topic, try to understand what you are doing and why you are doing it as much as possible. Sometimes it helps to know why things work, within certain limitations. I am not suggesting that you go through the proof of every theorem we discussed as this would be impractical and impossible since many of the theorems we discussed have very technical and complicated proofs. What I am saying, however, is try to at least have a cursory idea of what is happening.

(b) Go through the old homeworks; rework some of the problems on topics with which you may be struggling. It might be a good idea to work some problems in your text that I did not assign if you feel that you need extra practice with a particular topic. It is a bad idea to memorize problems. The purpose of the final is to serve as a metric for me to assess what you know and what you are capable of doing with that knowledge, not your ability to copy.

(c) Do not wait until the last minute to study! Moreover, do not let the fact that I am letting you use a ‘cheat sheet’ dupe you into believing that there is no need to study. From my own experience, I have found that the only students who benefit from such things are those students who arduously study and prepare.

3 Topics to Study

Here is a brief outline of topics that will be on the final. When applicable, I have tried to include the appropriate section in your text.

3.1 First Order Equations

(a) Separable Equations (Section 1.4)

(b) Basic Theory; Classification (i.e. order, linearity) of Equations; Existance and Uniqueness of First Order Equations

You should at least know how to apply the existance/uniqueness theorem that we discussed in class. Here it is, for reference:

Theorem
Let R be an (open) rectangle in the plane and let (x_0, y_0) be a point in the interior of R. Suppose that $f(x,y)$ and $\dfrac{\partial f}{\partial y}$ are continuous in R. Then there exists a unique local solution to the IVP

$$\begin{cases} y' = f(x,y) \\ y(x_0) = y_0 \end{cases}$$

(c) First Order, Linear Equations (Section 1.5)

(d) Bernoulli Equations; Homogenous Equations; Substitution Methods (Section 1.6)

(e) Exact Equations (Section 1.6)

You should know how to test for exactness and how to solve.

3.2 Higher Order Equations

(a) Basic Theory

Essentially, you should know the superposition principle for homogeneous linear equations (Pg. 149), linear dependence/independence, using the Wronskian to test for linear independence, and how to construct the general solution to a higher order linear equation (i.e. $y = y_h + y_p$ where y_h solves the homogeneous problem and y_p is a particular solution to the nonhomogeneous problem; see p. 161).

(b) Higher Order Linear Homogeneous Equations with Constant Coeficients (Section 3.3)

(c) Methods for Solving Nonhomogeneous Linear Equations of Higher Order

There are essentially two methods that we learned for solving such equations: undetermined coeficients and variation of parameters (see section 3.5 and your notes). You should know the limitations of each method and when to use one method over another.

(d) Cauchy-Euler Equations

These are equations of the form

$$a_n x^n y^{(n)} + ... + a_2 x^2 y'' + a_1 x y' + a_0 y = g(x)$$

To solve, we make the substitution $x = e^t$ so that $xy' = dy/dt$, $x^2 y'' = d^2y/dt^2 - dy/dy$, and so forth. This is the *only* higher order linear equation with nonconstant coeficients that we know how to solve.

3.3 Laplace Transforms

(a) Basic Ideas

Know the definition of the Laplace transform and how to use the definition to compute the transform of a function. Know the basic formulae for computing the transform of basic functions (i.e. the table on the front cover of your book; note that we didn't cover everything on that table and you will only be responsible for the things we discussed in class).

(b) The First and Second Translation Theorems

The First Translation Theorem simply says that if $f(t)$ has a Laplace transform given by $F(s)$, then $\mathcal{L}(e^{at}f(t)) = F(s-a)$. The Second Translation Theorem says that if $a > 0$, then $\mathcal{L}(f(t)H(t-a)) = e^{-as}\mathcal{L}(f(t+a))$ where $H(t-a)$ is the Heaviside function.

(c) Methods of Computing Inverse Transforms

(d) Convolution Theorem

(e) Applications of Laplace Transforms

Know how to use Laplace transforms to solve differential and integral equations (see the notes).

3.4 Applications

(a) Applications of First Order Equations

You should know the idea behind exponential growth and decay (p. 37-40) and Newton's Law of Cooling (pg. 40-41). You should also be familiar with the acceleration-velocity model discussed in class (Section 2.3).

(b) Spring-Mass Systems

Know how to derive and solve the equations for free undamped and damped motion of a spring mass system. Know how to determine if the system is overdamped, critically damped, or underdamped.

(c) LRC Circuits

3.5 Practice Problems

Here are some problems for you to practice solving. This is not a complete list of possible problems; the goal of this is for you to determine the method needed to solve each equation.

(a) $x^2y'' + 7xy' + 25y = 0$

(b) $xy' = y + \sqrt{x^2 - y^2}$

(c) $y'' + y = \csc^2 x$

(d) $(2xy^2 + 3x^2)dx + (2x^2y + 4y^3)dy = 0$

(e) $y^3y' = (y^4 + 1)\cos x$

(f) $x^2y' + 2xy - 5y^3 = 0$