18 March 2008
Math 242

Homework 9 - Second Order Equations

Directions: Please work these problems for homework. They will be due on Thursday, 3/25.

1. Work the following problems in your text: pg. 210 (1-6, 31-33,41) These problems concern variation of parameters and undetermined coeficients.

2. Consider the initial value problem

$$\begin{cases} xy'' - y' + 4x^3y = 2x^3 \\ y(\sqrt{\pi}) = -\frac{1}{2} \\ y'(\sqrt{\pi}) = 4\sqrt{\pi} \end{cases}$$

(a) Verify that $y_1(x) = \sin(x^2)$ is a solution to $xy'' - y' + 4x^3y = 0$.

(b) Use the fact that $y_1(x) = \sin(x^2)$ solves the homogenous problem in (a) to find the general solution to the homogenous problem.

(c) Use variation of parameters and the fundamental set you obtained in (b) to solve the nonhomogenous problem above.

(d) Use your general solution to the nonhomogenous problem you obtain in (c) to solve the initial value problem.

3. Solve the initial value problem

$$\begin{cases} y'' + 4y = g(x) \\ y(0) = 1 \\ y'(0) = 2 \end{cases}$$

where

$$g(x) = \begin{cases} \sin x & 0 \leq x < \frac{\pi}{2} \\ 0 & t \geq \frac{\pi}{2} \end{cases}$$

You want to solve this so that y and y' are continuous (*Hint*: Split the problem up separately on $[0, \pi/2)$ and on $[\pi/2, \infty)$ and then take limits to force the solution and its derivative to be continuous).