

Name (print legibly): **Solutions**

12 April 2007

Math 142

Quiz 8

Directions: You have 20 minutes to complete the quiz. Please show all relevant steps - if you show no work, I cannot give you partial credit. No calculators are allowed during the quiz. This quiz is worth 15 points.

1. (9 Points) Determine whether or not the following sequences converge or diverge. If so, find the limit.

(a) $a_n := 1 + (-1)^n$

(b) $b_n := n \sin\left(\frac{\pi}{n}\right)$

(c) $c_n := \frac{(-1)^{n+1}}{n^2}$

Solution. For (a), we note that $a_n = 0$ when n is odd and $a_n = 2$ when n is even. Because this series oscillates, it cannot converge. Part (b) was a homework problem (# 10 on page 633). We translate the sequence back to the reals and use L'Hopital's rule. That is:

$$\lim_{x \rightarrow \infty} x \sin\left(\frac{\pi}{x}\right) = \lim_{x \rightarrow \infty} \frac{\sin\left(\frac{\pi}{x}\right)}{1/x} = \lim_{x \rightarrow \infty} \frac{-(\pi/x^2) \cos\left(\frac{\pi}{x}\right)}{-(1/x^2)} = \pi \lim_{x \rightarrow \infty} \cos\left(\frac{\pi}{x}\right) = \pi$$

Thus, the limit of the sequence is π . Finally for (c), we note that

$$\frac{-1}{n^2} \leq \frac{(-1)^{n+1}}{n^2} \leq \frac{1}{n^2}$$

so by the squeeze theorem, $c_n \rightarrow 0$ as $n \rightarrow \infty$ because both the left and right hand sides of the above inequality go to zero.

2. (6 Points) Show that the sequence $a_n := \frac{n!}{3^n}$ is eventually increasing (or decreasing).

Hint: Consider a_{n+1}/a_n .

Solution. Observe that

$$\frac{a_{n+1}}{a_n} = \frac{n!(n+1)}{3^n \cdot 3} \cdot \frac{3^n}{n!} = \frac{n+1}{3} > 1 \quad \text{when } n \geq 3$$

That is, $a_{n+1} > a_n$ whenever $n \geq 3$ so that the sequence is eventually increasing.