

Name (print legibly): **Solutions**

22 March 2007

Math 142

Quiz 7

Directions: You have 20 minutes to complete the quiz. Please show all relevant steps - if you show no work, I cannot give you partial credit. No calculators are allowed during the quiz. This quiz is worth 15 points.

1. (7 Points) For each of the following rational functions, write out the partial fraction decomposition but DO NOT attempt to solve for the coefficients.

$$(a) \frac{3x^3 + x^2 - 6x + 11}{x^2(x^2 + x - 2)(x^2 + 4^2)} \quad (b) \frac{1}{x^3 + 2x^2 + 2x}$$

Solution. For (a), first note that $x^2 + x - 2 = (x + 2)(x - 1)$ and that $x^2 + 16$ is irreducible over the reals. Then, we have

$$\frac{3x^3 + x^2 - 6x + 11}{x^2(x^2 + x - 2)(x^2 + 16)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x + 2} + \frac{D}{x - 1} + \frac{Ex + F}{x^2 + 16}$$

For (b), first factor the denominator: $x^3 + 2x^2 + 2x = x(x^2 + 2x + 2)$. The quadratic formula tells us that $x^2 + 2x + 2$ is irreducible over the reals because its only roots are $x = -1 + i$ and $x = -1 - i$ (verify this by using the quadratic formula yourself). Thus

$$\frac{1}{x^3 + 2x^2 + 2x} = \frac{A}{x} + \frac{Bx + C}{x^2 + 2x + 2}$$

2. (8 Points) Compute $\int \frac{x-5}{x^2+2x-3} dx$.

Solution. First, we compute the partial fraction expansion of the integrand. We have

$$\frac{x-5}{x^2+2x-3} = \frac{x-5}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1} = \frac{(A+B)x + (3B-A)}{(x+3)(x-1)}$$

Thus, $A+B=1$ and $3B-A=-5$ so that $A=3B+5$ and $A+B=3B+5+B=1$ or $4B+5=1$ so $B=-1$. Then $A=1-B=2$. Then

$$\int \frac{x-5}{x^2+2x-3} dx = \int \left(\frac{2}{x+3} - \frac{1}{x-1} \right) dx = 2 \ln |x+3| - \ln |x-1| + C$$